Data acquisition and processing

Electronics involves many physical quantities like: voltage, current, resistance, energy, which are characterized by magnitude and by certain relationships between them. The quantitative estimation of the properties of these quantities is obtained through the measurement operation.

To measure, means to compare one quantity with another quantity, usually of the same kind, conventionally taken as a reference, which materializes the measurement unit. The measurement operation is expressed mathematically by the formula:

$$X = n \times U$$

Where: *X* – to be measured;
n – numerical value of the measured quantity;
U – reference value (measurement unit).

Any measurement process contains 4 main elements:
- the measurand;
- the measurement method;
- the measuring device;
- the standard.

The final result of any measurement is a number that characterizes, together with the unit, the measured quantity. In electrical engineering there are three types of measured quantities:

- constant quantities, for which the instantaneous value do not change regardless of the moment and the measurement time.
- stationary variable quantities, which can be characterized by instantaneous values, by the whole instantaneous values within a given time interval or by a global parameter such as:
  - the mean value: $X_{med} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} x(t)\cdot dt$;
  - the root mean square value: $X = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} x^2(t)\cdot dt}$;
  - the peak value: $x_m = \max_{t_1}^{t_2}|x(t)|$.
- non-stationary variable quantities, characterized by:
  - the instantaneous value at the certain time moment or a number of instantaneous values at the certain time moments;
  - the average value over a time interval;
  - all instantaneous values over the time period.

1. Errors in measurement science

Errors are present in every measurement result and they cannot be totally zeroed. If an experiment is well designed and carefully performed, the errors can be often reduced to a level where their effects are smaller than an acceptable level. Figure 1 presents a classification for the main categories of errors and describes some causes and methods of correcting them.

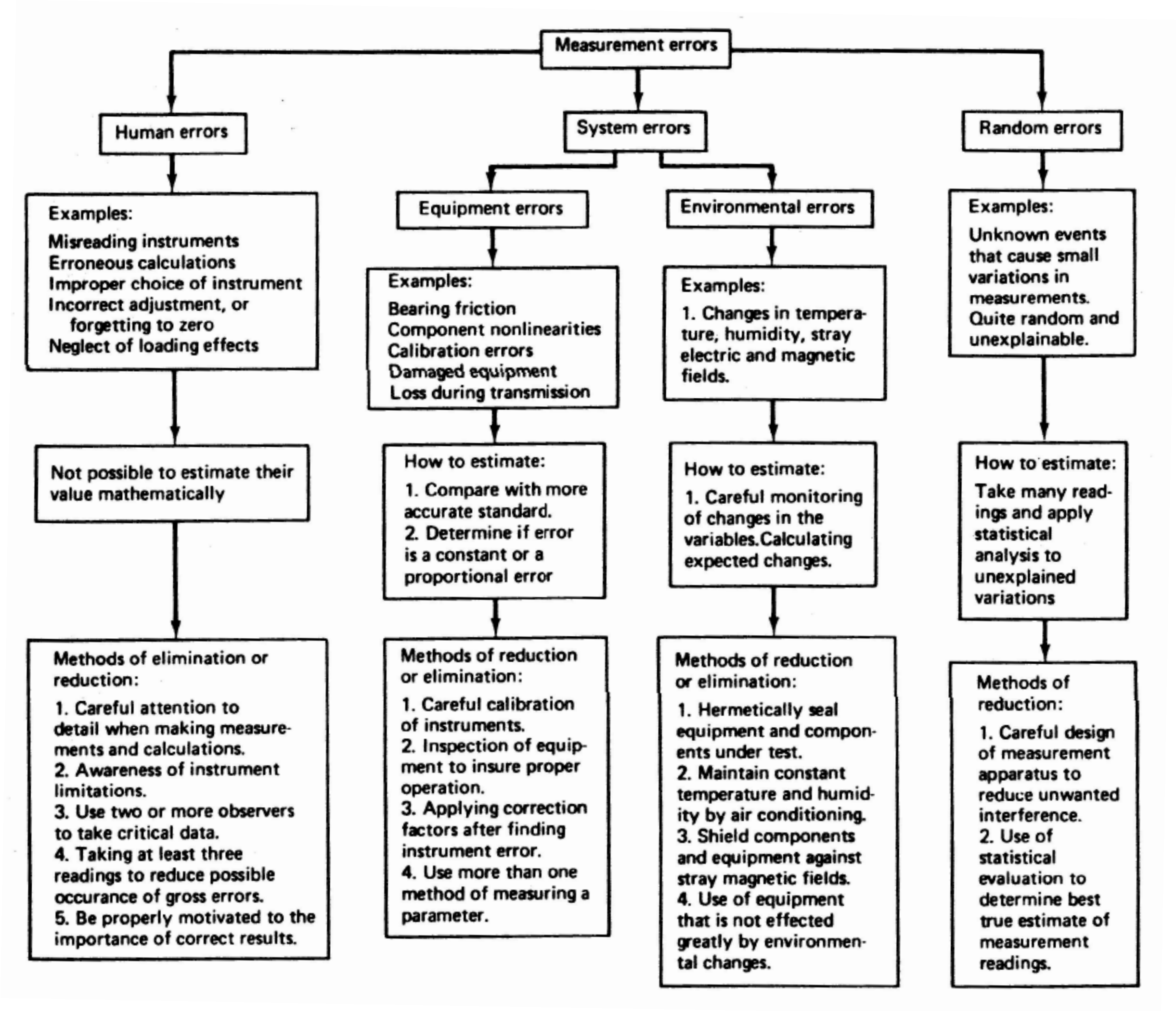


Figure1 Measurement errors: how to estimate, reduce, or eliminate them.

2. Instrumental precision and accuracy classes

The accuracy of a measurement system is the degree of closeness of measurements of a quantity to its actual (true) value. The instrumental precision of a measurement system is described by the instrumental error. This includes both, the systematic and the random error. For normalizing the tolerated errors (allowable) of the measuring instrument, the instrumental errors are divided into basic errors (intrinsic) and supplementary errors (influence).
Basic errors are the errors in reference conditions (environment conditions well defined), prescribed by regulations and standards.
Supplementary errors are generated by the variation of the influence parameters (environment). They are prescribed for the variation of each parameter separately, within their nominal ranges.
The instrument maximum permissible errors are given by one of the following forms:

The absolute error

It is the difference between the measured value of a quantity and its real value. It mainly used in some non-electric quantity measurements. Its expression is:

$$e = \pm\Delta X = \pm|X_m - X_e|$$

where e - absolute tolerated error, ΔX = a value expressed by the same units like the measured quantity, X_m is the measured value, X_e is is the real value.

The relative error (percent error)

The relative error gives an indication of how good a measurement is relative to the size of the thing being measured. Its expression is:

$$\varepsilon_r = \frac{|e|}{X_e} \cdot 100 \cong \frac{|e|}{X_m} \cdot 100 \ \ [\%]$$

where ε_r - the tolerated relative error;
x - the value of the measurand; x_e – the real value of the measurand;
$|e|$ - the absolute error modulus.

The referenced error (relative to a conventional value)
The referenced error gives an indication of how good a measurement is but relative to the size of a conventional value. It is computed with the formula:

$$\varepsilon_R = \frac{|e|}{X_C} \cdot 100 \ \ [\%]$$

where ε_R - the tolerated referenced error
$|e|$ - the absolute error modulus.
X_c – the conventional value.

X_c can be :
- the measurement upper limit of an instrument if the instrument has linear scale and 0 at the beginning;
- the nominal value of the measurand if the instrument has a nominal value;
- the scale length if the instrument has a nonlinear scale
- the sum of upper limits for instruments with 0 in the middle of the scale.

Combination of relative and referenced errors

This type of error is used for bridges, for compensators, for differential voltmeters, and for digital multimeters. It is expressed as a relative error:

$$\varepsilon_r = \pm\left(b + c \cdot \frac{X_C}{X_m}\right) \ \ [\%]$$

or as an absolute error:

$$e = \pm(b \cdot X_m + c \cdot X_C)$$

where ε_r - the tolerated relative error;
e - the tolerated absolute error;
X_m - the value of the measurand;
X_C - the upper limit of the measuring range of the instrument;
b, c - dimensionless positive numbers.

For digital multimeters the error is expressed as

$$e = \pm(b\% \ of \ reading + n \ digits)$$

The accuracy class reflects the instrument properties but it is not necessarily the measurement accuracy. The standardized values for the accuracy class are: 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1, 2, 5. The instruments for which it is used the relative of the referenced error, the accuracy class represents the tolerated base error in percents. For the instruments for which it is used the combination of relative and referenced error, the accuracy class is given by 2 numbers (b and c).
In the following figure are shown three ways of presenting the admissible errors:

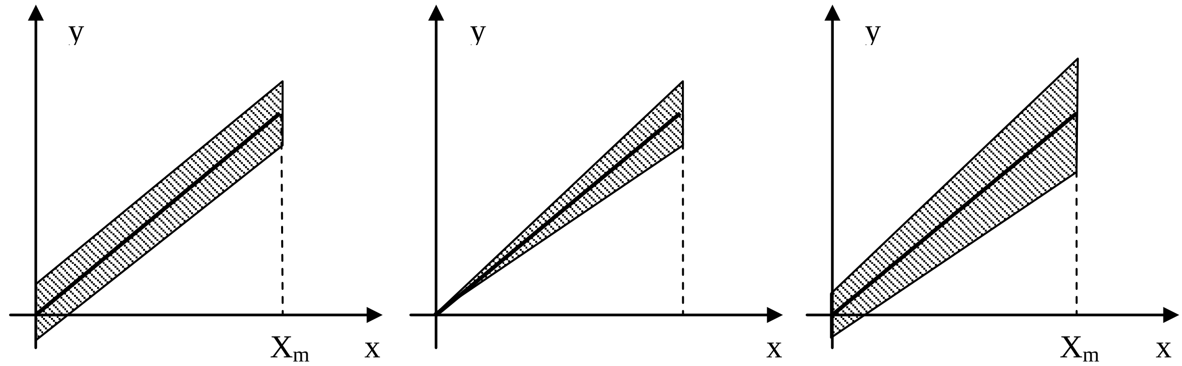


Figure 2 a) the absolute error is constant, b) the absolute error is variable, c) the absolute error has a constant part and a variable part

The following table gives examples of presenting the instrumental error and accuracy classes. The accuracy class is not the measurement error of the instrument. In most cases the absolute error is constant, but the relative error increases toward the lower scale limit:

$$\varepsilon_r = \frac{\Delta X}{X}\cdot 100 = \frac{\Delta X}{X_C}\cdot\frac{X_C}{X}\cdot 100 = \varepsilon_R\cdot\frac{X_C}{X}$$

Table 1

Mode of expression	Tolerated error	class index
Relative Error	$\varepsilon_r=\pm b\%$	b
Referenced error	$\varepsilon_R=\pm p\%$	p
Referenced error to the scale length	$\varepsilon_R=\pm p\%$	p
Combination of relative and referenced errors	$\varepsilon_r = \pm\left[b' + c\cdot(\frac{X_m}{x}-1)\right]$	b'/c

Example: If we use a voltmeter having the accuracy class c=1 (referenced error) and measurement is performed on the scale U_C=10V, the corresponding relative error is given by the above equation. If the measured value is U=5V, the relative error is $\varepsilon_r=c\cdot U_C/U=1\cdot 10/5=2\%$. If the measured value is U=8V, than the error is $\varepsilon_r=c\cdot U_C/U=1\cdot 10/8=1.25\%$. The relative error is minimum at the superior scale end where $\varepsilon_r=c\cdot U_C/U=1\cdot 10/10=1\%$ and it becomes equal with the referenced error. That is why it is indicated for such instruments to have the indication in the last third of the scale.

3. Experimental data collecting

Various physical quantities are not directly accessible to the human senses. In order to be measured, it has to be converted in another one by the measuring instrument. The optic message is the most convenient for the human operator: the indicator needle or the digital display. The measuring instrument can be actuated by the energy of the measured quantity or by an auxiliary energy.

The measuring method supposes the use of rational principles and procedures and of adequate instruments such that the result is the best approximation. The measurements are divided into:

- usual measurements
- precision measurements: - verification or calibration
  - for determining physical constants

Usual measurements do not require high accuracy and errors are not needed. They are performed where necessary using less sensible instruments based on deviation or differential methods. Usually one reading of the instrument indication is enough, but another measurement will ensure its correctness. If the indication has fluctuations, then many readings are necessary and the method for random error estimation must be used.

Precision measurements are also called lab measurements. They have high accuracy, the errors are estimated and the measured values are corrected. These measurements are usually performed in special climate rooms, electromagnetically shielded, using high sensitive instruments and comparison methods. They are use for scientific research, for calibration or verification of measuring instruments.

The calibration means the comparison of a measuring instrument with a standard, having the goal to do its gradation, the adjustment or the checking. *The gradation* is made during the fabrication process while the adjustment is made after repairing or if necessary in order to match the indication with the tolerable limits. *The verification* consists in checking if the instrumental errors are within the tolerated errors, according with its precision class. The calibration determines the corrections over the whole measuring range of the instrument.

The calibration/recalibration is periodically made according with the instrument accuracy (between 24 hours and 10 years). The ratio between the accuracy of the standard and the accuracy of the instrument is usual between 1/2.5 and 1/10, most frequent between 1/3 and 1/5.

The verification consists of 5 to 10 measurements in the main points of the measuring range. If the instrument has many measuring ranges, the instrument is verified in 3 to 5 equidistant points on each range. For low accuracy multimeters from the standard it is applied a value to produce a nominal deviation and the standard indication is used to calculate the error. For high accuracy multimeters, the values of the standard that produces the change of two consecutive numbers on the display are used to compute the errors.

The measurements for determining physical constants use functions depending on many variables that are measured. The indications of many instruments are used, 5 to 10 measurements from each one being necessary for the statistical analysis of the error and the constant calculation.

4. Data processing

Measurements are mainly affected by systematic errors or by random errors, while human errors can be avoided by careful attention to details, by taking many measurements or by using many observers. Sometimes usual measurements are strongly influenced by systematic errors, sometimes they have random errors are more important, while in other situations they have to be considered both.

4.1. Dominant systematic errors

When systematic errors are dominant, the instrument accuracy is decisive. It is specified in the instrument datasheet as the tolerated limit error. The result of the measurement has to be given as:

$$x = \pm x_m \pm \varepsilon ,$$

where x_m is the measured value (the value read on the instrument display) and ε is the uncertainty corresponding to the maximum error. For example if the instrument has the accuracy class 1 given as relative error, and the measured value is 3,2V, the measurement result is x=3,2±0.032V. This means that the measured value is located in the interval [3.168, 3.232]V.

For high accuracy measurements it has to be taken into account both, the systematic errors and the random errors. Thus, many measurement data have to be taken. From the instrument datasheet and from the measurement data, the limit error ±*a* have to be determined. Because between these limits the error can take any value, it can be considered as equiprobable (uniformly distributed probability). For so called rectangular distribution, the root mean square error is

$$\sigma = a/\sqrt{3}$$

Another possibility is estimate directly the limits a_i for each of the measurements and the root mean square error σ_i. The overall root mean square error can be afterwards calculated with

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(a_i)^2}{3}} .$$

The uncertainty of the systematic error is given by:

$$\varepsilon_{\max} = \pm t \cdot \sigma$$

where *t* can be find in the table in Appendix as for the Student method, choosing a confidence level *P*. This method is called randomize of the systematic error.

4.2. Dominant random errors

Random errors cause the indication fluctuation. Short term random errors generate fluctuations on the displayed value from one measurement to the other, while long term random errors need longer observation period. If random errors are dominant, for usual measurements, data coming from several readings must be processed with the Student method. The method should be applied for a small number of measurements (less than 10). If the number of measurements is very big, this distribution becomes the normal distribution (Gauss distribution).

The Student probability density function is given by:

$$p(t) = \frac{1}{\sqrt{n \cdot \pi}} \cdot \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n}{2})} \cdot \left(1 + \frac{t^2}{n}\right)^{-\frac{n+1}{2}} ,$$

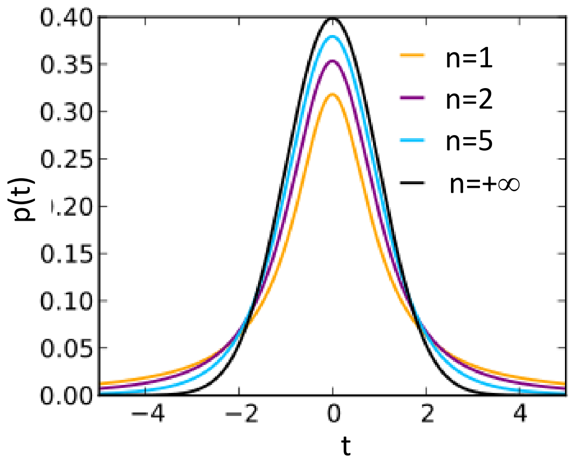
where *n* is the number of measurements (the number of degrees of freedom) and *Γ(n)* is the Euler function:

$$\Gamma(n) = \int_0^{\infty} y^{n-1} \cdot e^{-y} \cdot dy$$

Integrating *p(t)* from -∞ to t we get the Student distribution function:

$$F(t) = \int_{-\infty}^{t} p(u) \cdot du = \frac{1}{\sqrt{n \cdot \pi}} \cdot \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n}{2})} \int_{-\infty}^{t} \left(1 + \frac{u^2}{n}\right)^{-\frac{n+1}{2}} \cdot du$$

The functions *p(t)* and F*(t)* are shown in next figure.



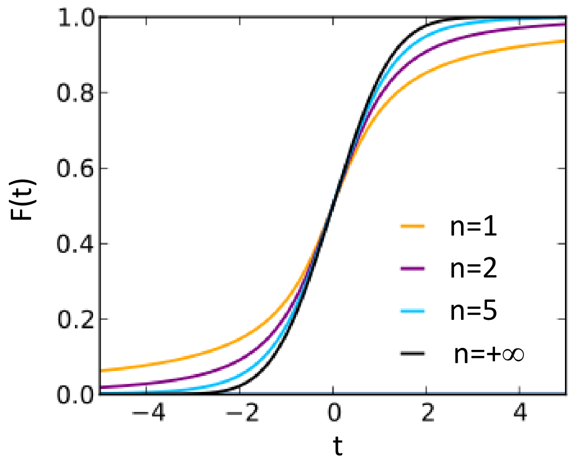


Figure 3 The probability density and the distribution function for Student distribution

The *t* variable is tabled (see appendix) as function of *n* (the number of measurements) and *P* (confidence level). The probability that the variable *t* is contained in certain interval (confidence level) is given by the following equation:

$$P(-t_i, t_i) = \int_{-t_i}^{t_i} p(t) \cdot dt$$

The result of a measurement affected by random errors is the mean of up to 10 measurements:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Because there are a small number of measurements, the mean calculated with the above equation is actually an estimation of the real mean and it is itself a random error. Thus, the estimation of the standard deviation is:

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

The deviation of the calculated mean from the real mean is:

$$\delta_{\bar{x}} = \pm \frac{t \cdot s}{\sqrt{n}}$$

Thus, the maximum error of a result from a string of measured data is

$$\delta_{Xmax} = \pm tS$$

In conclusion, in order to estimate the errors follow the following steps:

- choose a confidence level (ex *P*=99%),
- take 5 to 10 measurements,
- compute *S*,
- compute $S_{\bar{x}} = S/\sqrt{n}$,
- read *t* from the table in Appendix according to the number of measurements *n* and *P*
- find out the trust limits δ_{Xmax} and $\delta_{\bar{x}}$
- give the result as

$$x = \bar{x} \pm \delta_{Xmax} \quad \text{or} \quad x = \bar{x} \pm \delta_{\bar{x}}$$

Example: A set of 5 measurements is taken as it is shown in the table:

Nr. crt	1	2	3	4	5

I	153 mA	162 mA	157 mA	161 mA	155 mA

They are affected by random errors (the measured values are different). First we estimate the mean value:

$\bar{I}$= (153 + 162 + 157 + 161 +155)/5 =157 mA

Second we estimate the dispersion:

δ_{x1}	δ_{x2}	δ_{x3}	δ_{x4}	δ_{x5}
157-153=4mA	162-157=5mA	157-157=0mA	161-157=4mA	157-155=2mA

$$S = \sqrt{\frac{\delta x_1^2 + \delta x_2^2 + \delta x_3^2 + \delta x_4^2 + \delta x_5^2}{n-1}} = \sqrt{\frac{16+25+16+4}{4}} = \sqrt{\frac{61}{4}} = 3.9mA$$

Considering a confidence level of 95%, the variable t(95%, *n*=5)=2.78, the trust limit is:

$$\delta_{\bar{I}} = \pm\frac{tS}{\sqrt{n}} = \pm\frac{2{,}78 \cdot 3{,}9}{\sqrt{5}} = 4{,}84mA$$

And the final result can be given as: *I=157mA*±4,84*mA*

For estimating the random error in the case of high accuracy measurements, the Gauss method must be used.

The Gaussian (normal) distribution is based on some postulates:

- the results are affected only by random errors
- the deviation of the results are caused by *n* random factors
- the causes of these errors are independent
- the probability of positive errors is equal with that of negative errors
- large errors have the same probability of small errors.

The probability density of the normal distribution is:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\cdot\sigma^2}}$$

where μ is the mean of the results and σ is the standard deviation of random variable

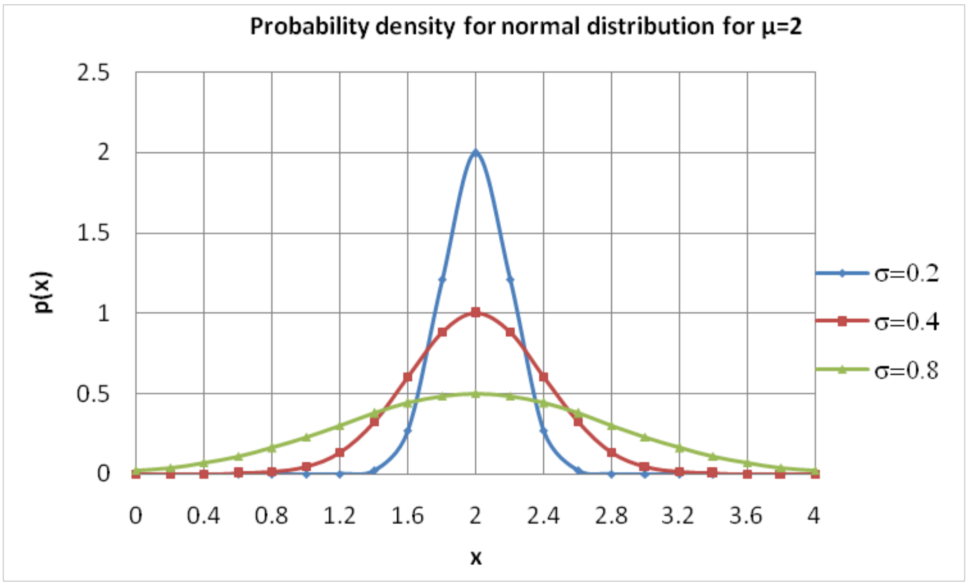


Figure 4 The probability density for Gauss distribution

It is called the normal law because the area under the curve is equal with 1:

$$\int_{-\infty}^{\infty} p(x)\cdot dx = 1$$

The probability that a measured data is located between x_1 and x_2 is the area under the curve between x_1 and x_2:

$$P(-x_1 < x < x_2) = \int_{-x_i}^{x_i} p(x)\cdot dx = \int_{-x_i}^{x_i} \frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(x-\mu)^2}{2\cdot\sigma^2}}dx = \sqrt{\frac{2}{\pi}}\cdot\Phi(z)$$

where $z=(x-\mu)/\sigma$, and $\Phi(z)$ is the Laplace function.
The mean and the dispersion for n measurements can be computed with:

$$\mu = \frac{\sum_{i=1}^{n} x_i}{n} \text{ and } \sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}}$$

The maximum error of any measurement is:

$$\delta_{Xmax} = \pm z\cdot\sigma$$

and the mean error is:

$$\delta_{\mu} = \pm\frac{z\cdot\sigma}{\sqrt{n}}.$$

In conclusion, in order to estimate the errors using the Gauss distribution follow the following steps:

- choose a value for z (ex z=1 – P=68% for usual measurements, z=2 – 95% for calibration measurements and z=3 – P=99,7% for precision measurements),
- take a large number of measurement data (>50),
- compute μ,
- compute the standard deviation $\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}}$
- compute the standard deviation of the mean $\delta_{\mu} = \pm\frac{z\cdot\sigma}{\sqrt{n}}$,
- give the result as

$$x = \mu \pm \delta_{Xmax} \text{ or } x = \mu \pm \delta_{\mu}$$

4.3. Combined effect of systematic and random errors

If a measurement is affected by both systematic and random errors that are quantified as ±x (systematic errors) and ±y (random errors) we have to express the combined effect of both types. One way of expressing it would be to sum the two separate components of error, resulting:

$$e = \pm(x+y)$$

However, a more usual way is their quadratic combination:

$$e = \pm\sqrt{x^2+y^2}$$

The result must be given as:

$$x = \mu \pm e$$

4.4. Aggregation of errors from separate measurements system components

A measurement system, often consists of several separate instruments, each of which being subject to errors. Therefore, what remains to be investigated is how the errors associated to each measurement instrument combine together, so that the total error can be estimated.
Let's consider the general case, when the result depends on n measurement variables:

$$y = f(x_1, x_2, x_3, ...x_n)$$

The total differential of f is:

$$dy = \frac{\partial f}{\partial x_1}\cdot dx_1 + \frac{\partial f}{\partial x_2}\cdot dx_2 + ... + \frac{\partial f}{\partial x_n}\cdot dx_n = \sum_{i=1}^{n}\frac{\partial f}{\partial x_i}\cdot dx_i$$

In the most unfavorable condition all products in the sum above are positive. If we consider instead dx_i - Δx_i, the last equation becomes and we assume that all errors contributes in the same way:

$$\Delta y == \sum_{i=1}^{n} \left| \frac{\partial f}{\partial x_i} \right| \cdot \left| \Delta x_i \right|$$

Thus, the relative error results:

$$\frac{\Delta y}{y} == \frac{\sum_{i=1}^{n} \left| \frac{\partial f}{\partial x_i} \right| \cdot \left| \Delta x_i \right|}{f(x_{1,}, x_2, \ldots x_n)} = \sum_{i=1}^{n} \frac{\left| \frac{\partial f}{\partial x_i} \right| \cdot x_i}{f(x_{1,}, x_2, \ldots x_n)} \cdot \frac{\left| \Delta x_i \right|}{x_i}$$

The last equation is in fact:

$$\frac{\Delta y}{y} = d[\ln f(x_{1,}, x_2, \ldots x_n)] \quad ,$$

which is used in practice (logarithmic differential method).
In conclusion, the errors estimation method is the following:

- take a number of up to 10 measurements, calculate y_i and the the mean $\mu = \frac{\sum_{i=1}^{n} y_i}{n}$
- compute the root mean square errors for each variable: $s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$
- calculate the standard deviation: $s = \sqrt{\sum_{i=1}^{n} \left(\frac{df}{dx_i} \right)^2 \cdot s_i^{\,2}}$
- choose trust degree *P* (ex 95% or 99%) and find *t* as for Student distribution
- calculate the random error $\delta = \frac{t \cdot s}{\sqrt{n}}$
- find the limits for the systematic errors $\pm a_i$ and calculate $\sigma_i = a_i / \sqrt{3}$
- calculate $\sigma = \sqrt{\sum_{i=1}^{n} \left(\frac{df}{dx_i} \right)^2 \cdot \sigma_i^{\,2}}$
- calculate the total systematic error with $\varepsilon \ = \pm t \cdot \sigma$
- calculate the final total error: $e = \pm \sqrt{\varepsilon^2 + \delta^2}$
- the final result is $y = \mu \pm e$

4.5. Graphical representation of experimental data

Usually the measurement data is an unordered data set. For a convenient interpretation the histogram representation is used.
Follow the steps below:

- take n measurements x_i
- The range of the values has to be divided into intervals having the same length (grouping intervals). The Sturges' formula is used:

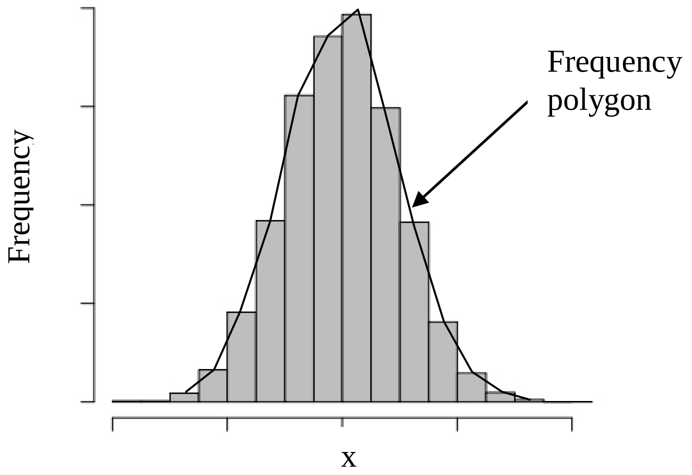
$$d = \frac{x_{\max} - x_{\min}}{1 + 3.22 \cdot \lg n}$$

- The value d is rounded to the nearest integer.

- Order the data in ascending order
- Calculate for each interval the central value
- Find the number of data n_k that fits into each interval (absolute frequency)
- Calculate the relative frequency $f_k=n_k/n$
- Put the data in the next table

Grouping intervals	Central value	Absolute frequency n_k	Relative frequency f_k

- Draw the histogram



Student name **Group**

Lab Worksheet

1. Study of accuracy class

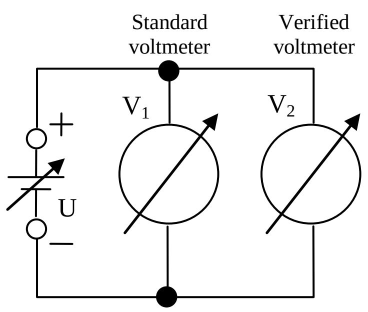
Check the datasheet or the instrument dial and complete the following table.

Device	**Marking**	**Accuracy class**	**Error type**	**Relative error**	**X_C**
Analog multimeter ……… $-V_{DC}$					
Analog multimeter ……… - A_{AC}					
Analog multimeter ……… - Ω					
Moving iron ampermeter					
Energy meter					
Standard resistor					
Digital multimeter .... V_{DC}					

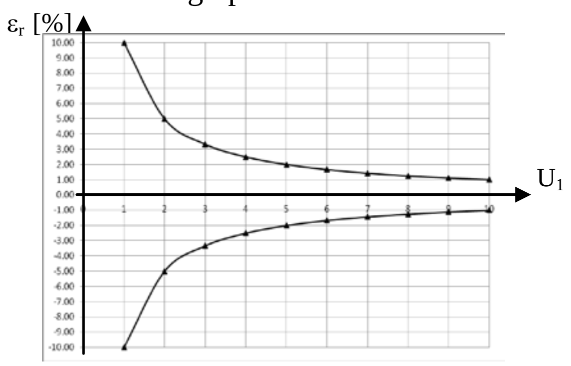
Conclusions:..
....
....

2. Checking the accuracy class:

a) Connect the instruments like in the following figure;
b) Collect the data and complete the table 1;
c) Draw the response of the verified voltmeter on the graph below.



a)



b)

Tabelul 1

Nr. crt.	U_{V1} [V]	U_{V2} [V]	ΔU [V]	ε_r [%]	ε_R [%]	Observations
1		1				V_1 type …………….
2		2				V_2 type ……………..
3		3				c =
4		4				$c_{exp.}$ =
5		5				
6		6				
7		7				
8		8				
9		9				
10		10				

Conclusions:……………………………………………………………………………
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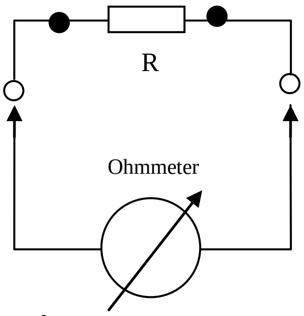
3. Measurements of quantities affected by random errors

a) Connect the instruments like in the following figure

b) Make 20 measurements by connecting the resistor R each time

Resistor type:



Ohmmeter type:

Tabelul 2

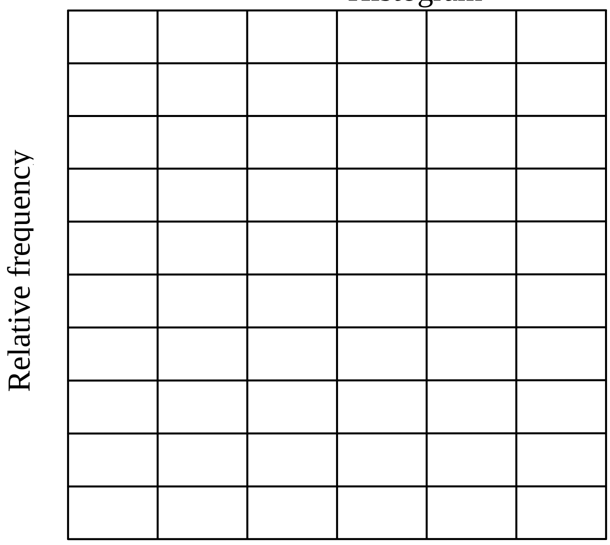
Nr. crt.	**R [Ω]**		**Nr. crt.**	**R [Ω]**
1			11	
2			12	
3			13	
4			14	
5			15	
6			16	
7			17	
8			18	
9			19	
10			20	

c) Evaluate the random errors using the Gauss and Student methods

Gauss	R_{mean}=	σ=	δ_μ=	R_m-δ_μ=	R_M+δ_μ=
Student	R_{mean}=	S=	$\delta_{\bar{x}}$=	R_m-$\delta_{\bar{x}}$=	R_M+$\delta_{\bar{x}}$=

d) Draw the histogram

Histogram



Resistance [ohms]

Conclusions:……………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………

Observation: The scematic diagrams must be completed with the instrument types used in the experiment.

Appendix 1

Table 1: t distribution

n \ P	50%	60%	70%	80%	90%	95%	98%	99%	99.5%	99.8%	99.9%
2	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
3	0.816	1.080	1.386	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
4	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
5	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
6	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
7	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
8	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
9	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
10	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
11	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
12	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
13	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
14	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
15	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
16	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
17	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
18	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
19	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
20	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
21	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291