# Vector measurements

Vector measurements allow finding out the components of a vector in one of its forms: cartesian or polar coordinates in real or polar plane. The result is given as 2 components: X and Y (Re and Im) or ρ and φ. Vector quantities are:

- impedance, admittance
- transfer functions (gain, attenuation)

The vector instruments are AC bridges (manual or automated), Q-meters, vector impedance meters, vector voltmeters. Automated AC bridges allow measuring the impedance or admittance components R, L and G, C. Manual bridges allow determining the components by calculation. Q-meters measure the vector by resonance. They measure the frequency, the capacitance and the quality factor. Automated Q-meters use the damped oscillation produced by loses in the resonant circuit. Vector impedance meters display the result in polar coordinates ρ and φ as functions of frequency. They maintain one quantity constant, voltage for admittance (the current is proportional with the admittance) or current for impedance (the voltage is proportional with the impedance). The phase shift between voltage and current and the ratio between the amplitudes are the modulus and the phase. Vector voltmeters have two voltage channels. One is the reference and the other is evaluated in respect with the reference.

## 1. AC bridges

In DC the resistances are measured with the Wheatstone bridge. The AC bridge is similar, but for balancing are necessary 2 variable elements:

$$\underline{Z}_x \cdot \underline{Z}_3 = \underline{Z}_2 \cdot \underline{Z}_4 \Rightarrow \begin{cases} Z_x \cdot Z_3 = Z_2 \cdot Z_4 & \text{(a)} \\ \varphi_x + \varphi_3 = \varphi_2 + \varphi_4 & \text{(b)} \end{cases} \quad \textbf{(1)}$$

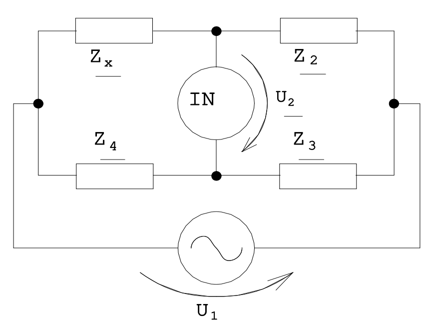



**Fig. 1**

One variable element balance the modules and the other the phases. From equations 1.1 and 1.b it results $z=Re(Z_x)+j\cdot Im(Z_x)$. The impedances depend on frequency, so the accuracy depends also on frequency. The sensitivity of an AC bridge is a complex parameter:

$$S = \Delta U_2/U_1 = F \cdot \delta \quad \textbf{(2)}$$

where: - $\Delta U_2$ is the unbalancing voltage that appear when one of the variable impedances is varied around the equilibrium

- $F$ is the bridge factor;
- $\delta = \delta R_i/Z_i$ or $\delta X_i/Z_i$ is the relative mismatch in one of the bridge arms.

The bridge sensitivity increases with the supply voltage, with the null instrument sensitivity, with the decrease of the impedances modulus or with the match of the four impedances ($Z_x \approx Z_2 \approx Z_3 \approx Z_4$).

## 2. Audio-frequency bridges (AF)

In audio frequency the components are modeled by two simple elements of circuit using a series or parallel configuration that approximates as good as possible their operation. There 2 types of bridges: for capacitors and for inductors, either for series equivalent configuration or parallel configuration.

### a) Audio frequency bridges for capacitors

Capacitors in audio frequency are measured mainly using the Sauty (series or parallel model) and Schering (high and low voltage and series model) bridges.

**Sauty bridges** compare the unknown capacitor with a standard capacitor in series or in parallel with a standard resistor (equivalent to the capacitor losses). The Sauty bridge for the series model is shown in figure 2 a), while the parallel model is shown in figure 2 b

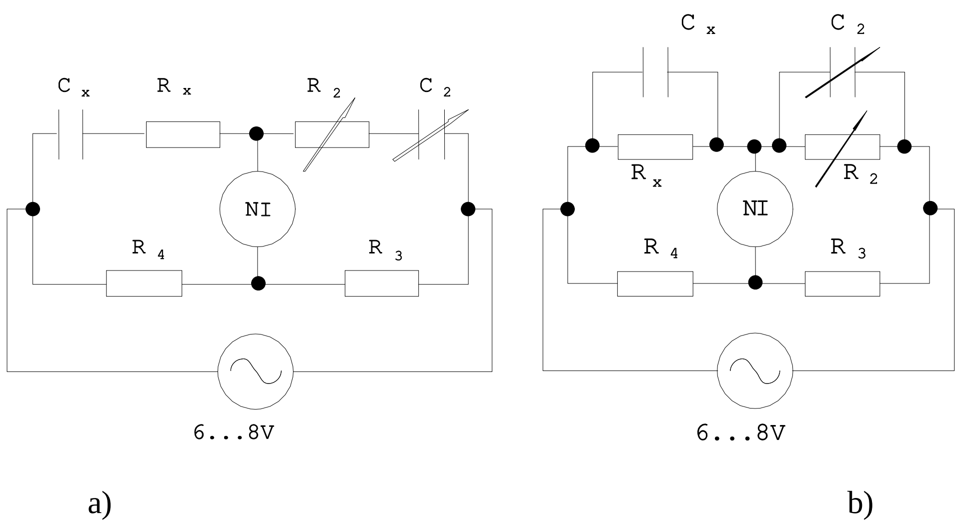



**Fig. 2**

The equations for the series configuration bridge are:

$$\left.\begin{aligned} C_x = C_2 \cdot \frac{R_3}{R_4}\ (a);\ \ R_x = R_2 \cdot \frac{R_4}{R_3}\ (b);\ \ tg\delta = \omega R_x C_x = \omega R_2 C_2\ \ (c) \\ Z_x = R_x - j\frac{1}{\omega\ C_x} = \frac{R_4}{R_3}\left(R_2 - j\frac{1}{\omega\ C_2}\right) \end{aligned}\right\} \quad (3)$$

The equations for the parallel configuration bridge are:

$$\left.\begin{aligned} C_x = C_2\cdot\frac{R_3}{R_4}\ (a);\quad R_x = R_2\cdot\frac{R_4}{R_3}\ (b);\quad tg\delta = \frac{1}{\omega C_x R_x} = \frac{1}{\omega R_2 C_2}\ (c)\\ Z_x = \frac{R_x}{1+j\omega j C_x R_x} = \frac{R_4}{R_3}\cdot\frac{R_2}{1+j\omega j R_2 C_2}\end{aligned}\right\}\quad \textbf{(4)}$$

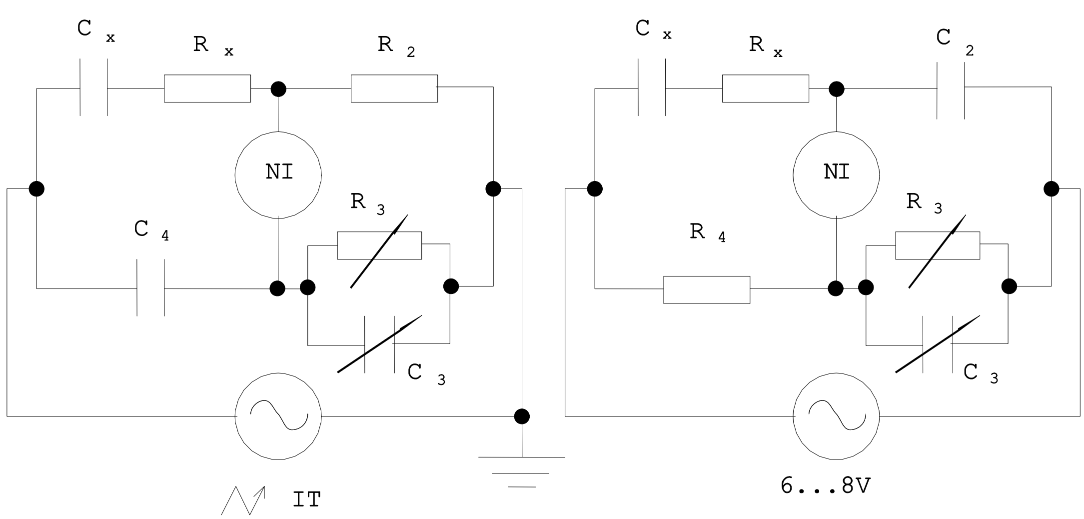
**Schering bridges** are presented in figure 3: high voltage in fig. a) and low voltage in fig. b). They are used only for the series model of the capacitor. Its main application is for measuring the insulating properties of electrical cables and equipment.

The equations under balance for HV bridge:

$$\left.\begin{aligned} C_x = C_4\cdot\frac{R_3}{R_2}\ (a);\quad R_x = R_2\cdot\frac{C_3}{C_4}\ (b);\quad tg\delta = \omega R_3 C_3\ (c)\\ Z_x = R_x\text{-}j\frac{1}{\omega C_x} = R_2\frac{C_3}{C_4}\left(1\text{-}j\frac{1}{\omega R_3 C_3}\right)\end{aligned}\right\}\quad \textbf{(5)}$$

The equations under balance for LV bridge:

$$\left.\begin{aligned} C_x = C_2\cdot\frac{R_3}{R_4}\ (a);\quad R_x = R_4\cdot\frac{C_3}{C_2}\ (b);\quad tg\delta = \omega R_3 C_3\ (c)\\ Z_x = R_x\text{-}j\frac{1}{\omega C_x} = R_4\frac{C_3}{C_2}\left(1\text{-}j\frac{1}{\omega C_3 R_3}\right)\end{aligned}\right\}\quad \textbf{(6)}$$



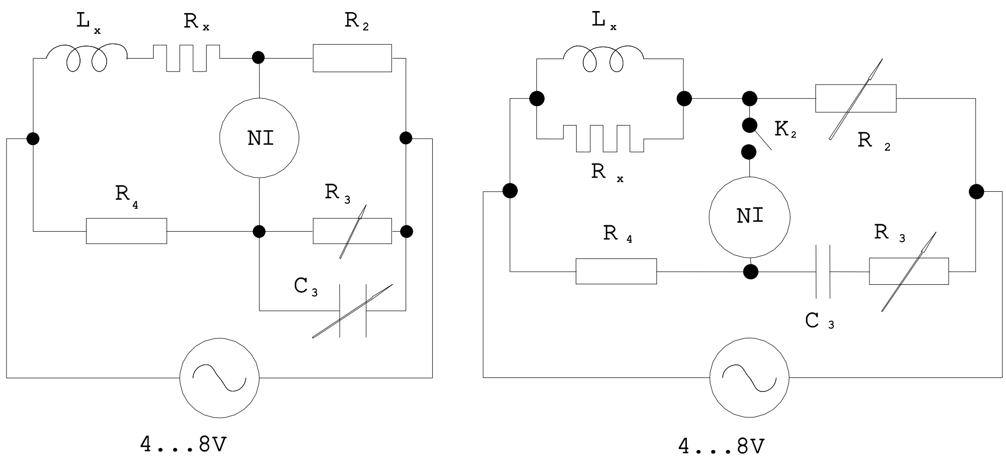

a) b)

**Fig. 3**

**b) Audio frequency bridges for inductances**

The most used bridges for measuring the inductances are the Maxwell-Wien for the series model and Hay for the parallel model.

The **Maxwell-Wien bridge** in fig. 4 a), is used for inductances with low quality factor, $1 \leq Q \leq 10$. The equations under balance are:




a) b)

**Fig. 4**

$$\left.\begin{aligned} L_x = C_3 \cdot R_2 \cdot R_4 \ (a); \quad R_x = R_2 \cdot \frac{R_4}{R_3} \ (b); \quad Q_x = \omega \cdot \frac{L_x}{R_x} = \omega R_3 C_3 \ (c) \\ Z_x = R_x + j\omega L_x = R_2 \frac{R_4}{R_3}(1 + j\omega C_3 R_3) = R_2 \frac{R_4}{R_3}(1 + jQ) \end{aligned}\right\} \quad \textbf{(7)}$$

The **Hay bridge**, fig. 4 b), is used for high quality factor inductances ($Q \geq 10$). The equations under balance are:

$$\left.\begin{aligned} L_x = R_2 \cdot R_4 \cdot C_3 \ (a); \quad R_x = R_2 \cdot \frac{R_4}{R_3} \ (b); \quad Q_x = \frac{R_x}{\omega L_x} = \frac{1}{\omega C_3 R_3} \ (c) \\ Y_x = \frac{1}{R_x} - j\frac{1}{\omega L_x} = \frac{1}{R_2} \cdot \frac{R_3}{R_4}\left(1 - j\frac{1}{\omega C_3 R_3}\right) \end{aligned}\right\} \quad \textbf{(8)}$$

The the series model can be transformed into the parallel model:

$$L_p = L_s \cdot \left(1 + \frac{1}{Q^2}\right) \ (a); \quad R_p = R_s \cdot \left(1 + Q^2\right) \ (b) \qquad \textbf{(9)}$$

For $Q > 5$ the two models have close values: $L_p \approx L_s$.

**Observations**

1. In AC it is not possible to reach the zero value at the balance, but a minimum value. This value reaches the lowest value when the bridge power supply has small distortions from a sinusoidal wave and

the disturbing signals are completely annealed. The null indicator is a frequency selective voltmeter or uses a synchronous detection on the frequency of the fundamental

2. The sensitivity to adjusting elements is changing during balancing. This asks for an alternate adjustment from the element that has the maximum influence until the minimum voltage is obtained.

3. Neglecting the sensitivity errors, the measurement errors are calculated starting from the equations under balance for the most unfavourable case.

### 3. Radio-frequency bridges (RF)

The most used bridge in RF is the Schering bridge. According to the model of the impedance there are two types: the Schering bridge for the series model and the Schering bridge for the parallel model: fig. 5 a) and b). The balance is achieved $C_1$, $C_3$ in 2 steps: without $Z_x$ and then with $Z_x$ connected in the bridge. The bridge for the series model is balanced in the first step with the connections $Z_x$ short circuited, while the bridge for the parallel model is balanced with the connections opened. When balanced, the series Schering bridge equations are:

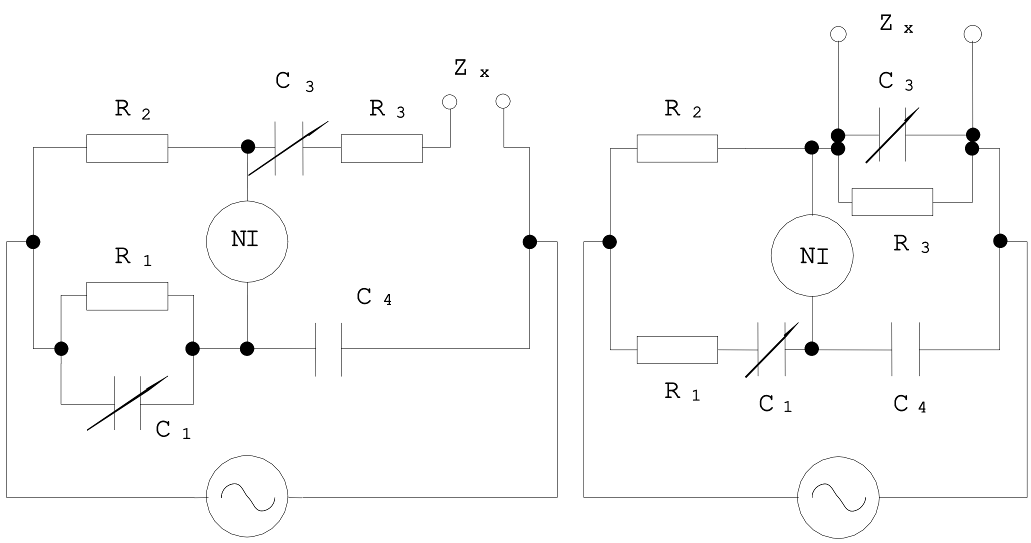



**Fig. 5**

- with the connections $Z_x$ short circuited:

$$\underline{Z}_{31} = \underline{Y}_{11} \cdot \underline{Z}_2 \cdot \underline{Z}_4 = \left(\frac{1}{R_1} + j\omega C_{11}\right) \cdot \frac{R_2}{j\omega C_4} = R_2 \frac{C_{11}}{C_4} + \frac{R_2}{j\omega R_1 C_4} \qquad \textbf{(10)}$$

- with the impedance $Z_x$ connected:

$$\left.\begin{aligned} \underline{Z}_x + \underline{Z}_{32} &= R_2\frac{C_{12}}{C_4} + \frac{R_2}{j\omega R_1 C_4} = \underline{Z}_{31} + R_2\frac{\Delta C_1}{C_4} \quad (a) \\ \underline{Z}_{31} - \underline{Z}_{32} &= \frac{1}{j\omega}\left(\frac{1}{C_{31}} - \frac{1}{C_{32}}\right) = j\frac{1}{\omega}\left(\frac{1}{C_{32}} - \frac{1}{C_{31}}\right) \quad (b) \end{aligned}\right\} \qquad \textbf{(11)}$$

The unknown impedance is:

$$Z_x = R_x + jX_x = R_2\frac{\Delta C_1}{C_4} + j\frac{1}{\omega}\Delta\left(\frac{1}{C_3}\right) \qquad \textbf{(12)}$$

When balanced, the parallel Schering bridge equations are:

- with $Z_x$ connections opened:

$$\underline{Y}_{31} = \underline{Z}_{11}\underline{Y}_2\underline{Y}_4 = \frac{1}{R_3} + j\omega C_{31} = \frac{C_4}{C_{11}R_2} + j\omega C_4\frac{R_1}{R_2} \qquad \textbf{(3)}$$

- with $Z_x$ connected:

$$\underline{Y}_x + \underline{Y}_{32} = \underline{Z}_{12}\underline{Y}_2\underline{Y}_4 = \frac{C_4}{C_{12}R_2} + j\omega C_4\frac{R_1}{R_2} = \underline{Y}_{31} + \frac{C_4}{R_2}\left(\frac{1}{C_{12}} - \frac{1}{C_{11}}\right) \qquad \textbf{(4)}$$

From the above equations (12) and (13) the admittance value is:

$$\underline{Y}_x = \frac{1}{\underline{Z}_x} = G_x + jB_x = \frac{C_4}{R_2}\cdot\Delta\left(\frac{1}{C_1}\right) + j\omega\cdot\Delta(C_3) \qquad \textbf{(5)}$$

The Schering bridges work for resistors, capacitances or inductances for frequencies up to 300 MHz.

## 4. Q-meter

The Q- meter uses the series resonance of a circuit. Figure 6 shows a schematic diagram of a Q-meter. At resonance, the voltage over $C_V$ is Q times the voltage $U_1$ – neglecting the resistance $r$ – the order of miliohms:

$$\omega^2 = \frac{1}{L\cdot C_v} \quad \textbf{(16.a)}; \qquad U_2 = Q\cdot U_1 \quad \textbf{(16.b)}$$

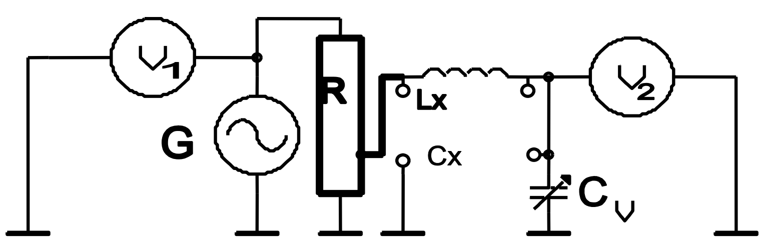


**Fig. 6**

Knowing the frequency, the capacity and measuring Q, the inductance L can be found:

$$L = \frac{1}{4\pi^2 f^2 C_v} \ (a); \quad Q = \frac{\omega L}{r} \ (b); \quad r = \frac{\omega L}{Q} = \frac{1}{2\pi f C_v Q} \ (c) \qquad \textbf{(17)}$$

$$Z_L = r + j\omega L = \frac{1}{2\pi f C_v Q} + j\frac{1}{2\pi f C_v} = \frac{1}{2\pi f C_v}\left(\frac{1}{Q} + j\right) \qquad \textbf{(18)}$$

The **digital Q-meter**, figure 7, is based on the damping of oscillations in an $L$, $r$, $C$ resonant circuit:

$$L\frac{d^2 i}{dt^2} + r\frac{di}{dt} + \frac{1}{C}i = 0 \qquad \textbf{(19)}$$

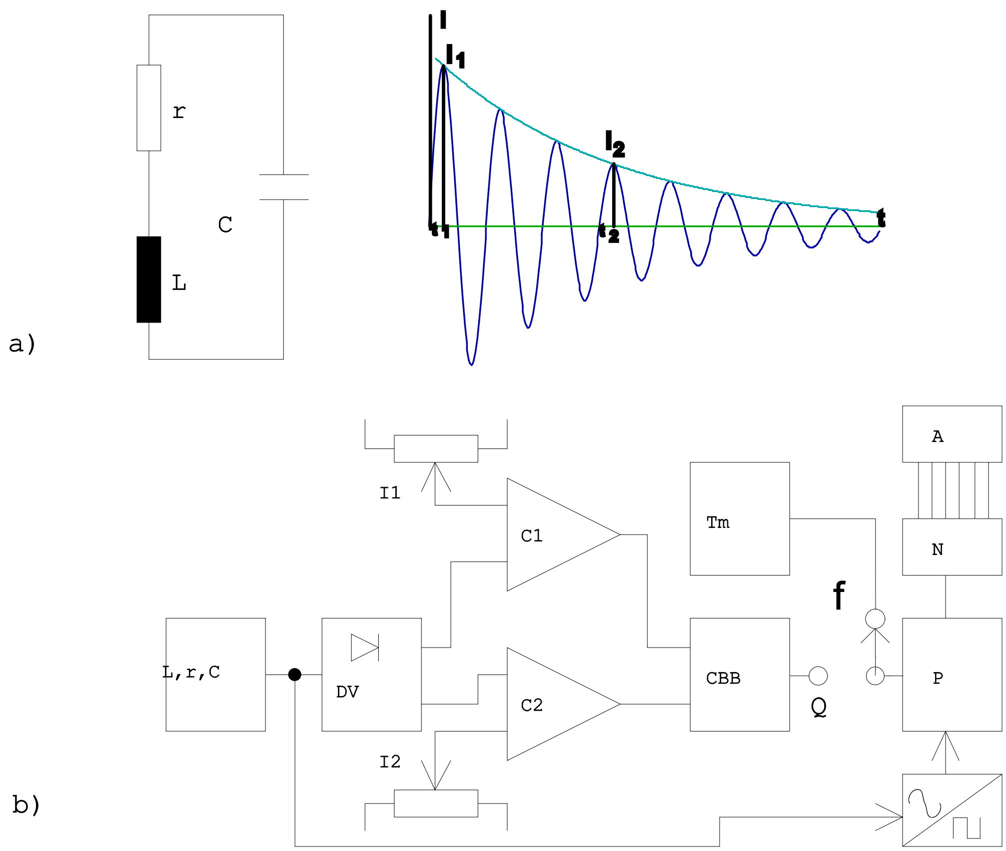



**Fig. 7**

The oscillating circuit is a second order system with a self oscillating angular frequency of:

$$\omega_0 = \sqrt{\frac{a_0}{a_2}} = \sqrt{\frac{1/C}{L}} = \frac{1}{\sqrt{L \cdot C}} \qquad \textbf{(20)}$$

and damping ratio of:

$$\xi = \frac{a_1}{2\sqrt{a_0 \cdot a_2}} = \frac{r}{2 \cdot \sqrt{L/C}} = \frac{r}{2 \cdot L \cdot \omega_0} = \frac{1}{2Q} \quad \textbf{(21)}$$

For $Q$>0,5 and $\xi$<1 – the circuit is underdamped, leading to sinusoidal oscillations as solution of equation (15):

$$i = Ae^{-\zeta\omega_0 t}\sin(\omega_0\sqrt{1-\xi^2}t+\varphi)\ ;\quad \xi \cdot \omega_0 = \frac{\omega_0}{2Q} = \frac{r}{2L} \quad \textbf{(22)}$$

Choosing the initial moment $t_1$, resulting:

$$i = I_1 \cdot \exp\left(-\frac{r}{2L}t\right)\cos\left[\omega_0\left(1-\frac{1}{2}\xi^2\right)t\right] \quad \textbf{(23)}$$

$$t_2 - t_1 = \frac{2L}{r}\ln K = \frac{2Q}{\omega_0}\cdot\ln K = N \cdot T = N\frac{2\pi}{\omega} \quad \textbf{(24)}$$

If $I_2=I_1/K$, the angular frequency is:

$$\omega = \omega_0\left(1-\frac{1}{2}\xi^2\right) \text{ because } \xi << 1,\quad \omega \cong \omega_0 \quad \textbf{(25)}$$

$$N = \frac{Q}{\pi}\cdot\ln K \quad \textbf{(26)}$$

Choosing $I_2$ such that $lnK = \pi$, equation (26) becomes: ***N=Q*** **(27)**.

The schematic in figure 7b) contains, besides the oscillating circuit, there is a peak detector that detects the levels $I_1$ and $I_2$ and drive a flip-flop (FF) that opens or closes the gate P. The pulses triggered from the damped oscillation pass through the gate and are counted by a counter, the final counted value N being displayed. Switching S on the position f the pulses are counted for given time interval (1s) and the frequency is measured $T_m = N_f \cdot T$ **(28)** $\Rightarrow$ $N_f = T_m/T = T_m \cdot f$ **(29)**. The instrument can have 2 counters to perform both measurements in the same time like in figure 8.

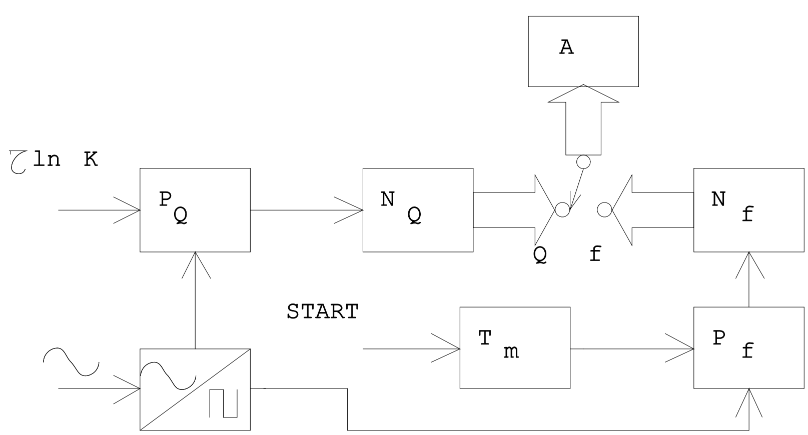



**Fig. 8**

Having the values $C$, $f$ and $Q$, the inductance components can be computed:

$$L = \frac{1}{4\pi^2 f^2 C} \text{ (a)}; \quad r = \frac{2\pi f L}{Q} = \frac{1}{2\pi f C Q} \text{ (b)}; \quad \underline{Z}_L = \frac{1}{2\pi f C}\left(\frac{1}{Q}+j\right) \text{ (c)} \qquad \textbf{(30)}$$

The accuracy of this kind of Q- meters is around 1- 2 % for Q and 0,1 - 0,2% for f. The display error has to be added ± 1/N.

## 5. Vector impedance meter

This is an instrument that gives the measurement result in polar coordinates (modulus and phase). The measurement can be performed for a single frequency or for a frequency band. The block diagram of a digital vector impedance meter in shown in fig. 9:

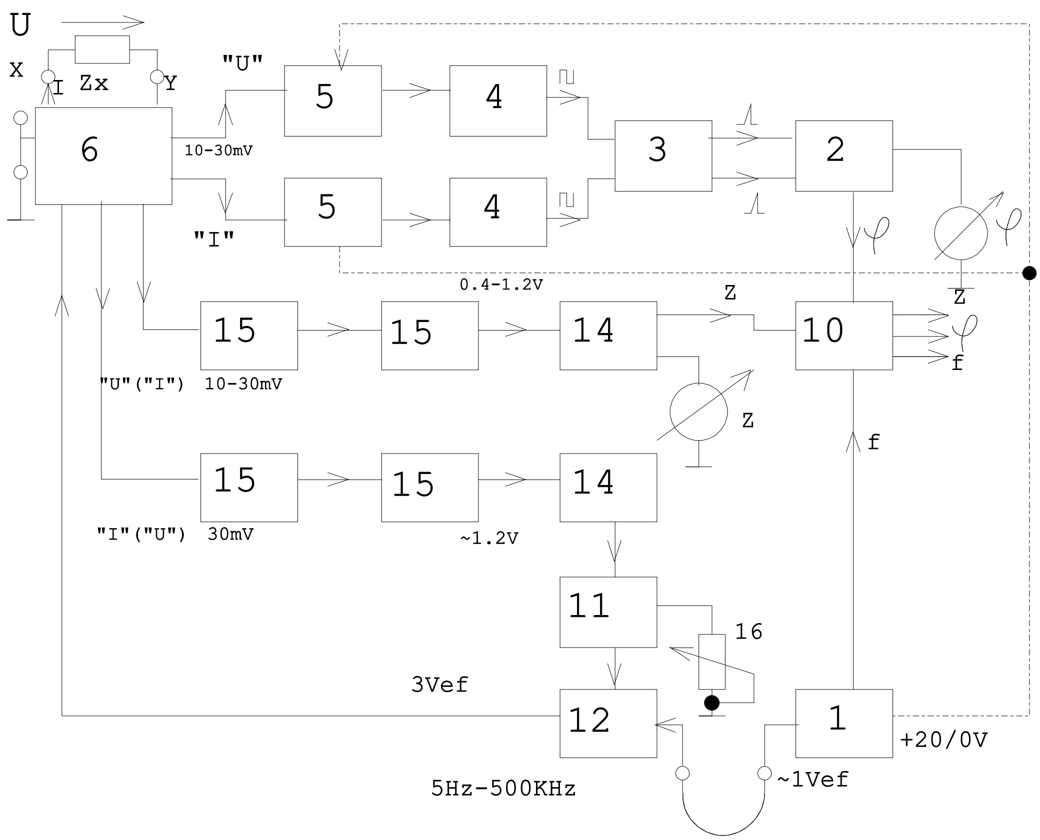



**Fig. 9**

1= signal generator; 2= phase detector; 3= pulse shaper; 4= limiter; 5= filter; 6= output stage; 10= analog outputs; 11= level stabilizer; 12= programmable amplifier; 14= detector; 15= amplifier; 16= calibration; Z= modulus display; φ = phase display.

The measuring mode is different for small impedances (under 1 kΩ) in respect with high value impedances. For the first mode the current through the impedance is kept constant, the voltage being the measure of the impedance/admittance, while in the second case the voltage is kept constant, the current being the measure of the impedance/admittance.

- for impedances with small value:

$$\underline{Z}_x=\frac{\underline{U}_x}{\underline{I}_x}=\frac{U_x}{I_x}(\arg\underline{U}_x-\arg\underline{I}_x)\ \text{(a)};\ \ \underline{I}_x=I=ct.\ \text{(b)};\ \ \underline{Z}_x=\frac{U_x}{I}\arg\underline{U}_x\ \text{(c)} \qquad \textbf{(31)}$$

- for impedances with high values:

$$\underline{Y}_x = \frac{1}{\underline{Z}_x} = \frac{\underline{I}_x}{\underline{U}_x} = \frac{I_x}{U} arg\, \underline{I}_x \qquad \textbf{(32)}$$

The constant current is between 1mA and 3 μA and the constant voltage between
3V and 3 mV according to the measuring range. The phase shift between the constant quantity and the measuring signal is measured with a phase meter with flip flop and displayed in degrees. The modulus is displayed on a linear scale in the first case and inverted in the second case. For certain values of the frequency (33), the reactive components are given in equations (35) and (36), where ρ is the measured modulus and φ is the measured phase. When the phase is not 0 or ± 90$^o$ the results must be corrected with *sinφ* and *cosφ*:

$$f = \frac{10^i}{2\pi}\ \text{(a)},\ \ \text{i} = 1,2...\ \text{(b)} \qquad \textbf{(33)}$$

$$\omega = 2\pi f = 10^i\ \text{(a)};\quad \sin\varphi = \pm 1\ \text{(b)} \qquad \textbf{(34)}$$

$$L = \frac{\rho}{\omega} = \rho\cdot 10^{-i} \qquad \textbf{(35)}\ ; \qquad C = \frac{1}{\rho\cdot\omega} = \frac{10^{-i}}{\rho} \qquad \textbf{(36)}$$

$$\underline{Z}_x = R+jX = \rho\cos\varphi + j\rho\sin\varphi\ \text{(a)};\ \ \text{Q} = \text{tg}\varphi\ \text{(b)};\ \ \text{tg}\delta = \text{ctg}\varphi\ \text{(c)} \qquad \textbf{(37)}$$

$$L = 10^{-i}\rho\sin\varphi \qquad \textbf{(38)} \quad \text{or:} \quad C = 10^{-i}\frac{1}{\rho\sin\varphi} \qquad \textbf{(39)}$$

The measurement errors are in the range of 5% for modulus and ± 6$^o$ for phase.

## 6. Vector voltmeter

The vector voltmeter measures 2 voltages, one being the reference and both having a constant frequency. The measurement is performed through synchronous sampling. If the sampling period is chosen as:

$$T_E = m\cdot T + \frac{T}{n}\ ,\ m,n \in N\text{-}\{0\} \qquad \textbf{(40)}$$

after sampling, the signal's frequency will be:

$$T_{FI} = m\cdot n + 1 \qquad \textbf{(41).}$$

In fig. 10 the principle of synchronous sampling is presented. Fig 11 presents the block diagram of a vector voltmeter. Such instrument is used to measure voltages between 1mV and 1V and frequencies up to 1GHz. Because the signals are simultaneously sampled the phase shift between them is kept with enough accuracy. The channel A is the reference signal that is used to synchronize the vector voltmeter. The intermediary frequency is 20kHz. The amplitudes and the phase shift of the two signals are converted into digital, processed by the processor and displayed.

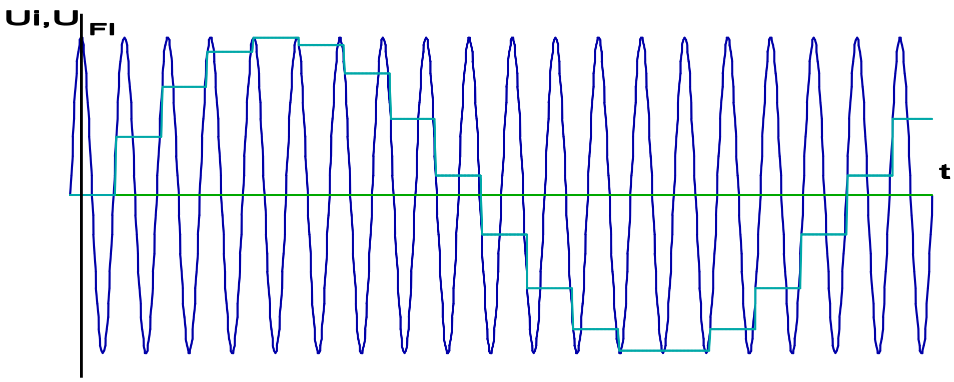



**Fig. 10**

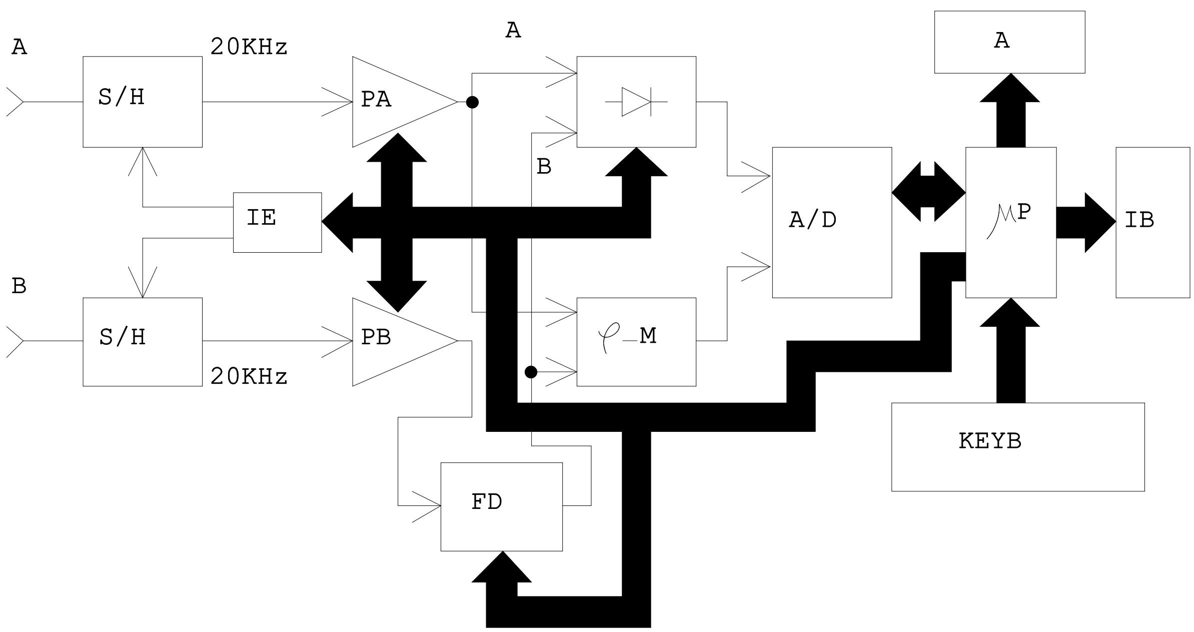



**Fig. 11**

### 1. Use of the vector voltmeter to measure the impedance

In order to measure the impedance, a coaxial test fixture like in fig. 12 must be used. The instrument can be interfaced with a digital frequency to compute $L_s$, $R_s$ respectively $C_p$, $R_p$. The unknown impedance can be computed with:

$$\frac{\underline{U}_A - \underline{U}_B}{\underline{Z}_x} = \frac{\underline{U}_B}{\underline{Z}_0} \qquad \textbf{(42)}$$

$$\underline{Z}_x = R_x + j X_x = R_0 \left( \frac{1}{U_B / U_A} \cos\varphi - 1 \right) - j R_0 \frac{1}{U_B / U_A} \sin\varphi \qquad \textbf{(43)}$$

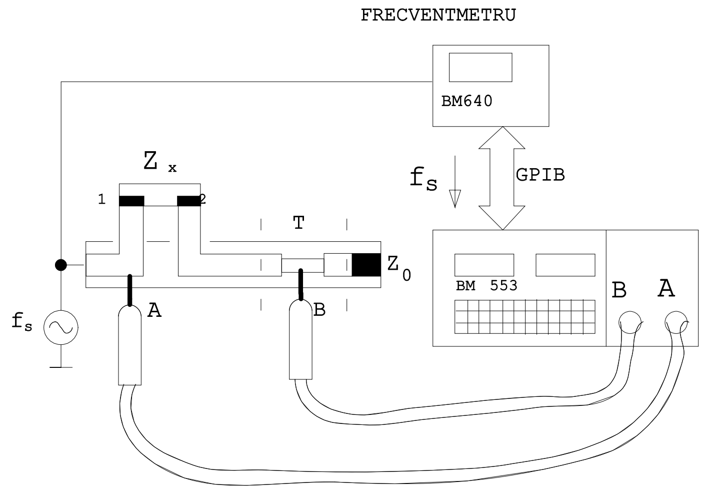



**Fig. 12**

## WORKS TO BE COMPLETED IN THE LAB

1. Various instruments for vector measurements will be studied.

2. R, L and C components will be measured at low and RF frequencies. Parallel and series models correspondence will be verified

3. Based on the results and limit errors, the methods will be characterized by showing the advantages and disadvantages.

Lab No. 5 Vector measurements

Group Student name Date

## Lab no. 4 *Vector measurements*

**Experimental work**

1. Inductances and capacitances measurement using the vector impedance meter, figure 1.

**Table 1**

| No. | f [Hz] | φ [°] | \|Z\| [Ω] | $C_x$ [nF] | $R_x$ [Ω] | $L_x$ [mH] | Obs. |
|---|---|---|---|---|---|---|---|
| 1 | 159 | | | | | | C= |
| 2 | 1591 | | | | | | |
| 3 | 15915 | | | | | | |
| 4 | 159155 | | | | | | |
| 5 | 159 | | | | | | R= |
| 6 | 1591 | | | | | | |
| 7 | 15915 | | | | | | |
| 8 | 159155 | | | | | | |
| 9 | 159 | | | | | | RC series<br>R=<br>C= |
| 10 | 1591 | | | | | | |
| 11 | 15915 | | | | | | |
| 12 | 159155 | | | | | | |

Conclusions:

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Figura 1


Figura 2

**2.** Measurement of passive components in LF with the automatic RLC bridge Agilent, figure 2.

**Table 2**

| No. | $C_S$ [nF] | $R_S$ [Ω] | $\|Z_S\|$ [Ω] | $C_P$ [nF] | $R_P$ [Ω] | $\|Z_P\|$ [Ω] | Obs. |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | RC parallel<br>R= C= |
| 2 | | | | | | | C= |
| 3 | | | | | | | L= |

Conclusions:

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**3.** Inductance measurement in RF with Q-meter, figure 3.

**Table 3**

| No. | f [kHz] | $C_V$ [pF] | Q | $C_{parazit}$ [pF] | $R_X$ [Ω] | $L_X$ [μH] | $\varepsilon_L$ | $\varepsilon_R$ | Obs. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 140 | | | | | | | | |
| 2 | 280 | | | | | | | | |
| 3 | 140 | | | | | | | | |
| 4 | 280 | | | | | | | | |

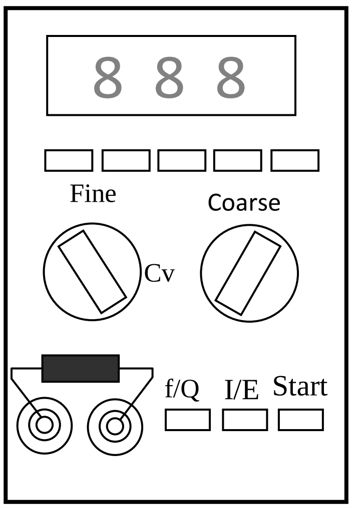


Figura 1

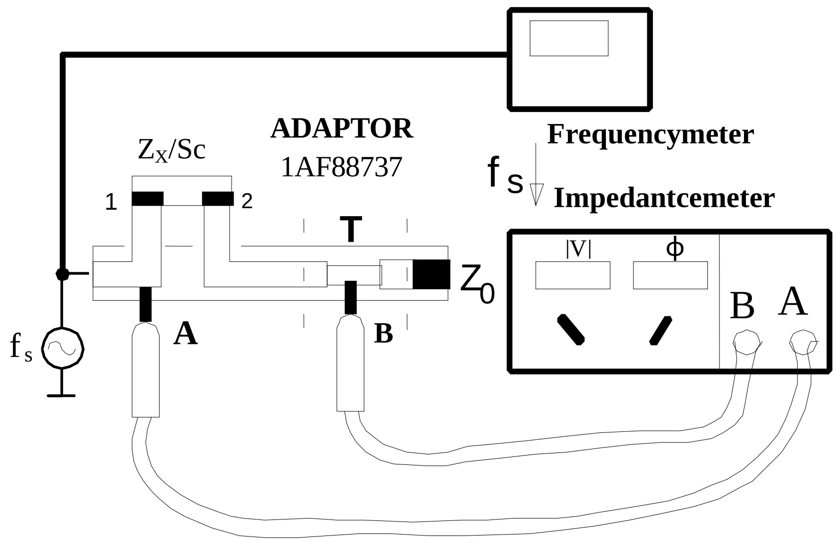


Figura 2

Conclusions:

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**4.** Passive component measurement in RF using the vector voltmeter, figure 4.

**Table 4**

| No. | f [MHz] | Φ [°] | $V_A$ [mV] | $V_B$ [mV] | $V_B/V_A$ | $Z_L$ [Ω] | $C_X$ [pF] | $R_X$ [Ω] | $L_X$ [nH] | Obs. |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1,591 | 0 | | | | $\underline{Z}_0$=50 | | | | |
| 2 | 1,591 | | | | | $\underline{Z}_0+\underline{Z}_X$ | | | | |
| 3 | 3,183 | 0 | | | | $\underline{Z}_0$=50 | | | | |
| 4 | 3,183 | | | | | $\underline{Z}_0+\underline{Z}_X$ | | | | |

Conclusions:

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