Frequency Estimation Based on Variable Frequency Resolution Concept

Andrei Sebastian Ardeleanu, Codrin Donciu
Faculty of Electrical Engineering
"Gheorghe Asachi" Technical University of Iaşi
Iaşi, România
a.ardeleanu@ee.tuiasi.ro

Abstract— **Frequency estimation is used in various applications, such as wireless communications, acoustic and speech processing, sonar and radar measurements. The classic approach uses the Fourier Transform, which estimates the frequencies at fixed points in frequency domain. This may cause the Fourier analysis to miss the real values of a signal's frequencies, also known as the picket fence effect. To overcome this issue, different interpolation techniques can be used, resulting in an increase in the estimate process. This paper presents a new algorithm of estimating the frequencies of a signal, using a variable frequency resolution concept.**

Keywords-frequency estimation; discrete fourier transform; variable frequency resolution

I. INTRODUCTION

The classic approach of estimating the harmonic frequencies is by using the Fourier Transform. Frequency estimation is performed in signal processing, with utility in various applications such as wireless communications, acoustic and speech processing [1-4], sonar and radar [5-6] measurements. Over time, various methods of frequency estimation were proposed in order to improve the accuracy and time of execution. These can be classified in methods performed in time domain and methods performed in frequency domain. Time domain methods give good results for fundamental frequency estimation, but when an estimation of harmonic frequencies is desired, frequency domain methods are more adequate to use.

Frequency domain methods use the fact that periodic signals present harmonic components which can be identified as peaks placed at equal distances in the magnitude spectrum. These peaks can be located in the spectrum obtained with the help of the Discrete Fourier Transform (DFT). An improvement of DFT was brought by Cooley and Tukey who implemented the Fast Fourier Transform (FFT), which decreased the computational demand needed to calculate the frequency spectrum [7]. Shortly after, the two major drawbacks of their canonical versions, the spectral leakage and the picket fence effect, have been revealed, and their reduction has become the aim of many scientific papers.

The spectral leakage is caused by the algebraic operation performed by FFT onto signals with non-integer number of periods, and the only way to completely eliminate the leakage effect is the coherent sampling.

But working with coherent signal is practicable only if prior information regarding its periodicity is available. On the other hand, signal periodicity represents one of the signal parameters revealed by FFT algorithm. Thus, an iterative algorithm must be implemented to solve this challenge, a large number of operations being necessary. The first approximation of signal's periodicity obtained after FFT computing is used to adjust the samples number of a second FFT computing, from coherences point of view, and so on.

However, coherent sampling is difficult to achieve in real world applications and, since spectral leakage is almost inevitable, the windowing technique is used. This technique multiplies the samples sequence of investigated signals with a predefined signal sequence, called window. A regular window has a unit gain at centre of sequence and decrease gradual to zero at both extremities, its shape suppressing the negative effect of non coherent sampling, observable at extremities of signal sequence. A signal sequence without a window is, in fact, a rectangle windowed one.

Many windows have been developed or improved and reported in scientific papers [8-12] and their improvement still continues. A general overview of different windows proprieties can be found in [13] and the influence of windows on detection of closely spaced spectral components are provided in [14] where twelve different windows types were investigated.

According to the acquisition parameters, the spectrum components are calculated at fixed points in frequency domain given by the frequency resolution Δf :

$$\Delta f = \frac{f_S}{N} \tag{1}$$

where f_s represents the sampling frequency and N is the number of samples of the sequence subject for processing.

Since the value of the frequency resolution is fixed, most of the times, the peaks obtained through DFT will not correspond to the real frequencies of the investigated signal. The real frequency will be between two DFT values and its estimation will be influenced by the resolution bias error (ε_b) whose value satisfies:

$$-\frac{f_S}{2N} \le \varepsilon_b \le \frac{f_S}{2N} \tag{2}$$

This effect is known as the picket fence effect and it similar to looking at a mountain range through a picket fence. Only for certain condition the real peaks will be observed. This effect leads, in general, in obtaining peaks that are too low in level and valleys that are too high.

To reduce the estimation error, different interpolation methods in combination with DFT were used to estimate the harmonic frequencies. For the two values, between which the real frequencies are present, an interpolation function is applied and the maximum of the interpolated spectrum will correspond to the investigated frequency. Interpolation using the parametric cubic convolution [3], parabolic interpolation [15] and other interpolation algorithms [16-19] were used to estimate the harmonic frequencies.

In this paper an improvement for the DFT algorithm for frequency estimation is presented. Increasing the resolution, thus decreasing the Δf parameter, in order to obtain a better precision of amplitude and frequency values, leads to an increase of the computational volume when obtaining the frequency spectrum. For the on-line signal processing operations, the processing time is an essential parameter, whose minimizing represents a permanent preoccupation.

Increasing the resolution, in terms of increasing the volume of operations, becomes efficient in calculating the Fourier Transform in the vicinity of spectral components and inefficient in the intervals between the spectral components. Optimizing these aspects can be achieved by a method which uses a variable resolution for calculating the frequency spectrum. In the following chapters, such a method will be presented.

II. MATHEMATICAL MECHANISM FOR ACHIEVING THE VARIABLE RESOLUTION FREQUENCY SPECTRUM

The mechanism of obtaining the frequency spectrum of a signal is based on a mathematical equation based on multiplication of two sinusoids:

$$\sin(A)\cdot\sin(B)=\frac{1}{2}\cos(A-B)-\frac{1}{2}\cos(A+B) \tag{3}$$

where $\sin(A)$ is a signal whose frequency A is wanted to be determined and $\sin(B)$ is a reference signal of frequency B. If from the investigated signal N samples are taken at a f_s sampling frequency, to obtain the frequency spectrum, a sweep of the reference signal's frequency with a step of Δf , given by (1), is required so that for each frequency a corresponding spectral component is obtained.

While the reference signal's frequency is different from the one of the investigated signal, the mean of the product signal will have a value close to zero. This value will be zero if the number of samples N for which the Fourier Transform is calculated is an integer number of periods of the investigated signal. If the reference signal's frequency is in the $(A-\Delta f, A+\Delta f)$ range, the mean of the product signal will take a non-zero value, event corresponding to identification the investigated signal's frequency.

A better understanding of the Fourier Transform can be achieved by analyzing the two signals obtained from the multiplication of the investigated signal with the reference signal. Out of these two signals, the first one, difference signal, has a stronger influence on the result due to its smaller frequency which gives a greater mean value. This can be observed in Fig. 1 where the mean values of the two signals are represented for an investigated signal of 100 Hz and Δf = 0.6 Hz. As the frequency of the reference signal is further from the one of the investigated signal, the frequencies of the two signals will be greater and therefore their mean will tend to zero. Conversely, if the reference frequency is getting closer to the investigated one, the mean values of the two signals will increase, especially of the difference signal. Since N is fixed, when the reference frequency takes a value inside the $(A-\Delta f, A+\Delta f)$ range, the difference signal will have less than a period, so its mean will attain a maximum. A maximum also will be attained from the difference of the two signals' means which will be used to estimate the frequency of the investigated signal.

Although is proportional with signal's amplitude, the mean value of the product signal calculated for a reference frequency located in the $(A-\Delta f, A+\Delta f)$ range is also influenced by the phase shift between the reference and the investigated signal. To eliminate this inconvenient an additional reference signal is used, shifted with 90° from the first one. If the first reference signal is $\sin(B)$ then the second will be $\cos(B)$. The configuration which uses two sinusoidal signals shifted by 90° allows obtaining the real amplitudes of the investigated signals and their harmonics (Fig. 2).

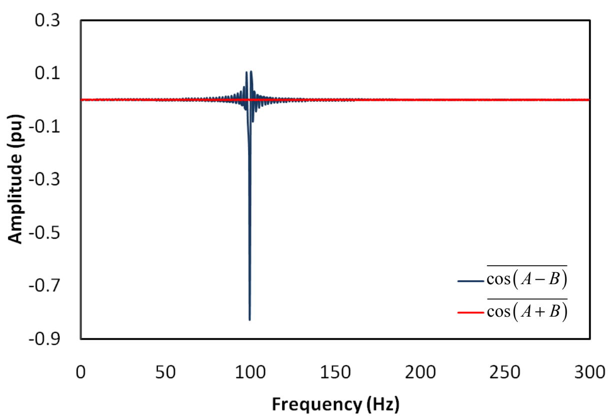

Figure 1. Mean values for the two signals.

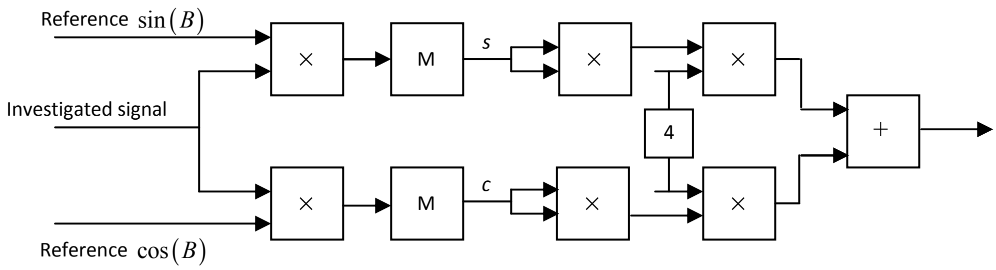

Figure 2. Fourier Transform algorithm block diagram.

The algorithm supposes sweeping the frequency of the two reference signals and multiplying them with the investigated signal. The two means of the product signals are squared, multiplied by four and afterwards summed. The result of these operations represents the Fourier Transform at the frequency point equal with the frequency of the two reference signals.

Since the algorithm suppose a sweep of the reference frequency with a Δf step, frequency estimation depends on the value of this step. If the investigated signal's frequency is f, the range of interest, which contains the useful information, is $(f-\Delta f, f+\Delta f)$. Reducing the size of this interval will result in obtaining a better estimate of the investigated frequency. Considering a sinusoid of 156 Hz and amplitude 1 sampled at a sample rate of 1 kHz, the frequency spectrums for different values of Δf are represented in Fig. 3.

From Fig. 3 it can be seen that increasing the resolution, Δf decreases, leads to a improvement in frequency estimation. Unfortunately an increase in the volume of operations is also observed, given the fact that more points have to be calculated in order to obtain the frequency spectrum. This inconvenient can be eliminated through a new technique which increases the resolution Δf in the area of interest and maintains it to a minimum value in rest.

This technique uses a variable resolution, $(\Delta f)_v$, which takes into account the previous amplitude of the frequency spectrum. If A is the amplitude of the spectral component, the variable resolution will be a function of A. This function has to be simple, with descending characteristics and when A tends to zero the variable resolution must tend to Δf.

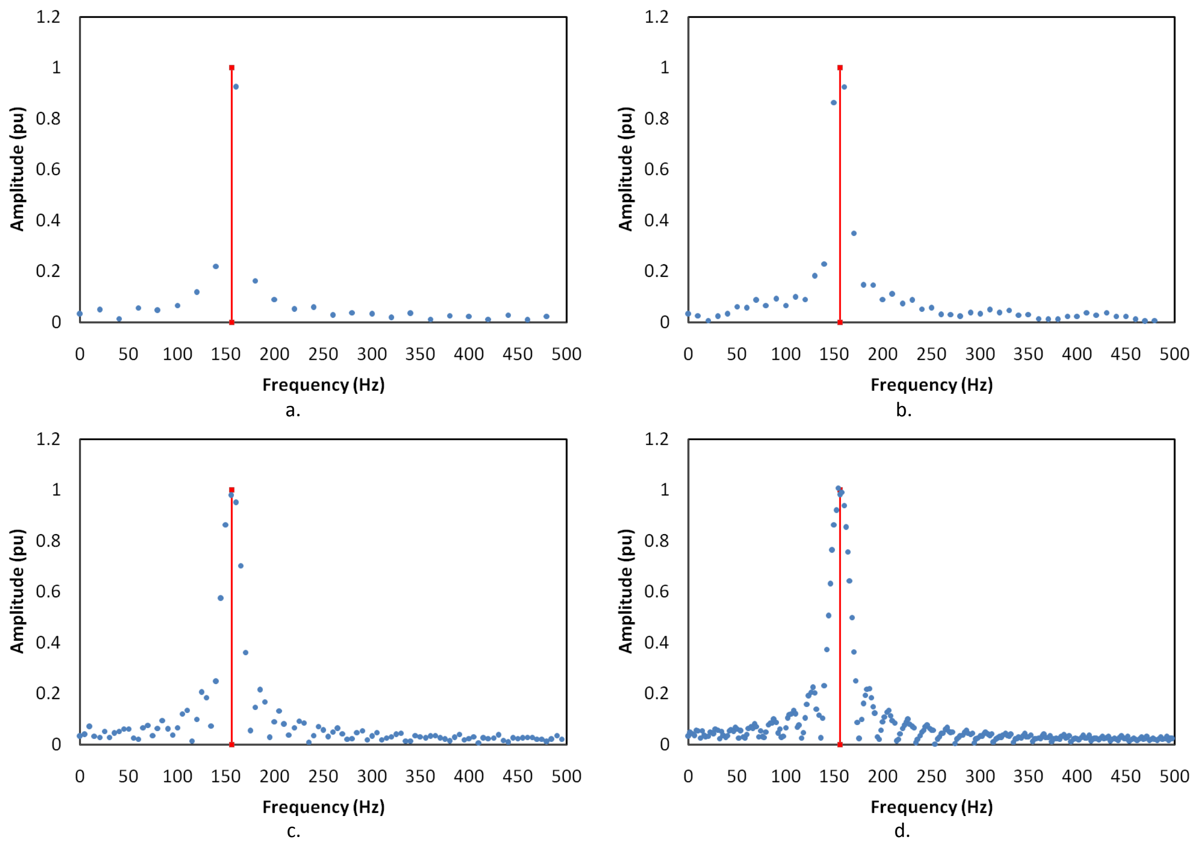

Figure 3. DFT for different values of Δf : Δf = 20 Hz (a), Δf = 10 Hz (b), Δf = 5 Hz (c) and Δf = 2 Hz (d).

Considering k an amplification factor, the function will be:

$$\left(\Delta f\right)_v = \frac{\Delta f}{1+kA} \tag{4}$$

or

$$\left(\Delta f\right)_v = \frac{\Delta f}{k^A}. \tag{5}$$

Applying (4) the resolution will have a close value to Δf when calculating the points that are not in the vicinity of the signal's frequency. When the amplitude increases, thus approaching to the signal's frequency, the resolution will decrease according to the chosen amplification factor, leading to a better approach to the signal's frequency. According to the values of the frequency resolution obtained with (4), the frequencies for every point are calculated. Estimating the signal's frequency corresponds to finding the highest value of the spectral components and taking its corresponding frequency.

III. EXPERIMENTAL RESULTS

Besides improving the frequency estimation, this algorithm also leads to an improvement in the amplitude and phase estimation of the investigated signal. Therefore, the experimental results are intended to present the improvements brought by this technique for every parameter by comparing it with the classical method. The following presumptions were taken into consideration when performing the algorithm simulation: sampling frequency f_s was set to a value of 200 kHz, 200 samples were taken for analysis (N) and the investigated signal is sinusoidal whose frequency varied between 1 kHz to 100 kHz with a step of 100 Hz. Also the amplification factor k was varied in order to determine the value that provides the best estimates.

In Fig. 4 the relative errors for the frequency estimation using the proposed algorithm with the amplification factor k taking values from 0 to 5 are presented. The results obtained for k = 0 correspond to the classic case when the frequency resolution is constant ($\left(\Delta f\right)_v = \Delta f$, see (5)). In Fig. 4a the values for the entire range can be observed, while in Fig. 4b a zoom of the error values between 10 kHz and 40 kHz is presented. From the zoom it can be clearly seen that the error values are considerable higher when the frequency resolution is maintained constant than when it is variable. This can be seen also when comparing the mean values of the relative errors. These values are 0.0116 pu for k = 0, 0.0019 pu for k = 1, 0.0017 pu for k = 2, 0.0024 pu for k = 3, 0.0017 pu for k = 4 and 0.0017 pu for k = 5.

For amplitude estimation, the values of the relative errors for the entire frequency range can be observed in Fig. 5a, while in Fig. 5b a zoom of these values for the frequency range between 50 kHz and 80 kHz can be observed. Likewise in frequency estimation it can be observed that the errors obtained for the constant resolution are higher than those obtained with a variable resolution. The mean values of the errors are: 0.1315 pu for k = 0, 0.0122 pu for k = 1, 0.0121 pu for k = 2, 0.0123 pu for k = 3, 0.0098 pu for k = 4 and 0.0091 pu for k = 5.

As previously mentioned, the better the frequency is estimated, the better the amplitude and the phase of the signal is also estimated. According to the algorithm, described in Fig. 2, the value of each spectral component represents the real signal's amplitude at the correspondent frequency. To estimate the phase ϕ, the arctangent of the rapport between cosine factor c and sine factor s (Fig. 2) is calculated:

$$\phi = \arctan\left(\frac{c}{s}\right) \tag{6}$$

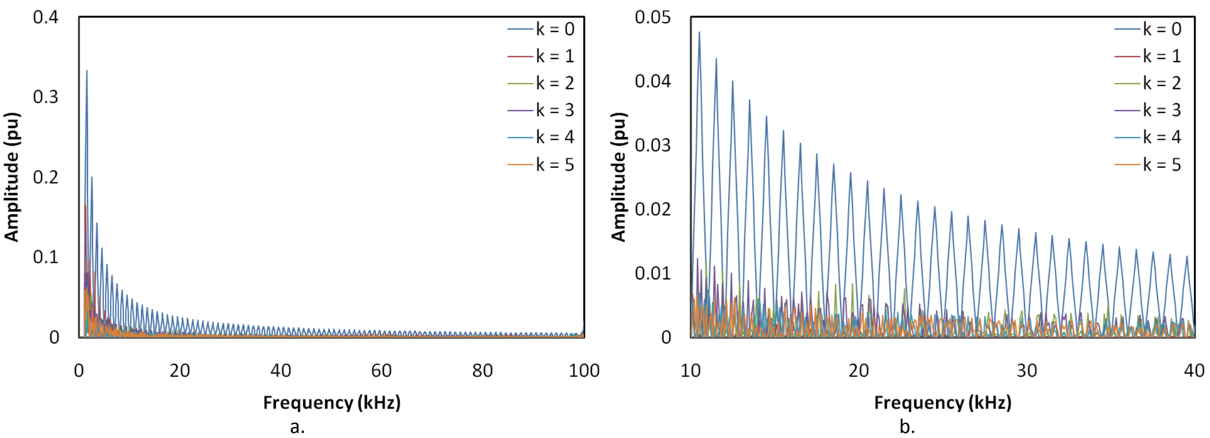

Figure 4. Relative errors of frequency estimation for k = 0 … 5: the whole frequency range 0 – 100 kHz (a) and a zoom between 10 and 40 kHz (b).

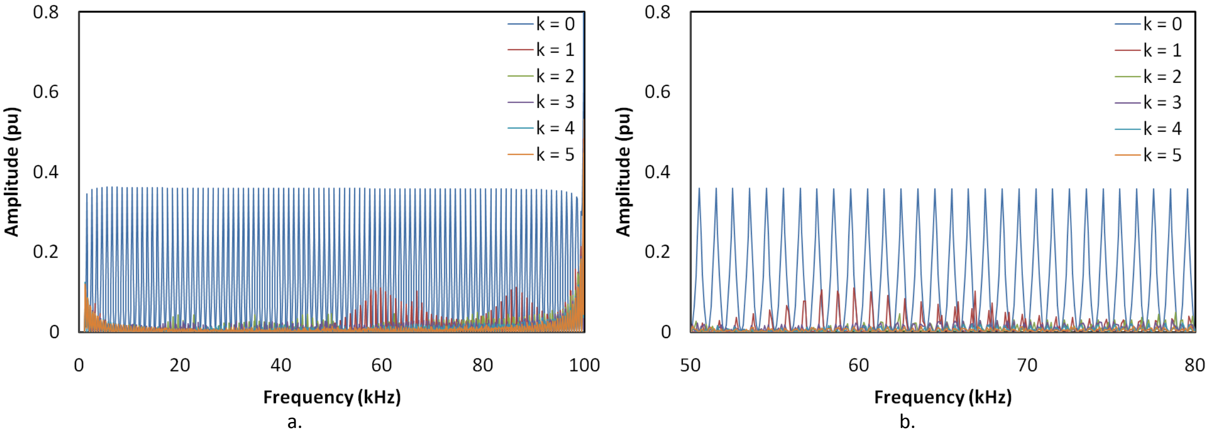

Figure 5. Relative errors of amplitude estimation for k = 0 … 5: the whole frequency range 0 – 100 kHz (a) and a zoom between 50 and 80 kHz (b).

The relative errors for the phase estimation can be observed for the whole frequency range in Fig. 6a and for a section of the frequency range between 50 kHz and 80 kHz in Fig. 6b. Also for the phase estimation can be observed that applying a variable frequency resolution results in smaller relative errors than applying a constant frequency resolution. The mean values of the errors are: 1.84811 pu for $k = 0$, 0.1043 pu for $k = 1$, 0.1450 pu for $k = 2$, 0.1815 pu for $k = 3$, 0.1391 pu for $k = 4$ and 0.1271 pu for $k = 5$.

As it can be seen, the variable resolution offers better results than the constant resolution when estimating the parameters of a sinusoidal signal. This is achieved although with the cost of an increase in the volume of operations the algorithm must perform. In order to evaluate this cost, the number of points needed to be calculated for each case was recorded. The representation of these points can be observed in Fig. 7. As expected, for $k = 0$, corresponding to a constant frequency resolution, the number of points remains constant. For $k > 0$, which corresponds to applying a variable frequency resolution, the number of points is slightly bigger and varies between certain limits. The mean values of the number of points are: 100 for $k = 0$, 102 for $k = 1$, 104 for $k = 2$, 106 for $k = 3$, 108 for $k = 4$ and 110 for $k = 5$. Analyzing these values it can be see that the increase in the number of points is quite small ranging from 2% for $k = 1$ to 10% for $k = 5$.

IV. CONCLUSIONS

In this paper, a new concept of variable frequency resolution for obtaining the frequency spectrum of a signal is presented. The purpose of this algorithm is to improve the accuracy of frequency estimation. This improvement also leads to an improvement of amplitude and phase estimation. The algorithm is simple, easy to implement and offers better results with a small increase in the number of operations. The accuracy of the algorithm can be further improved by applying an interpolation technique at points where the spectrum reaches its maximum.

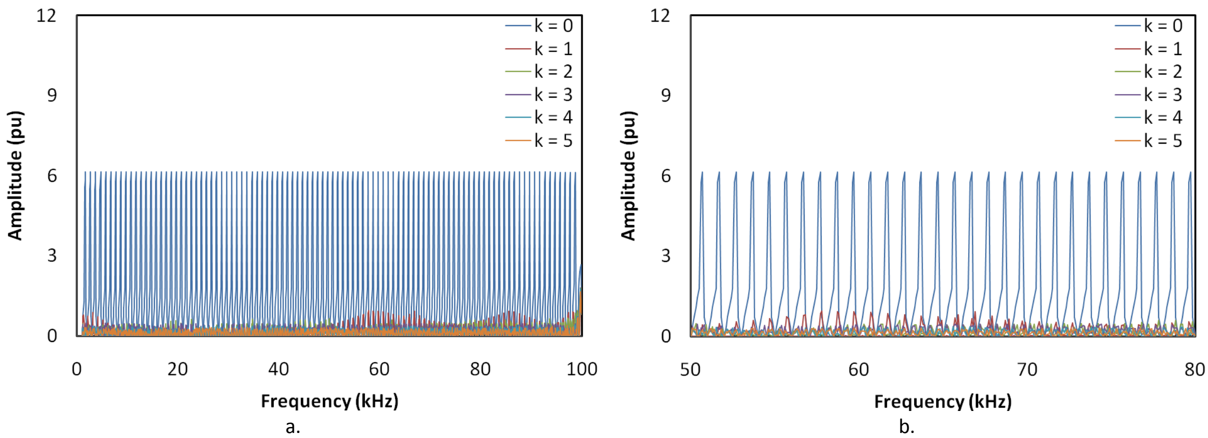

Figure 6. Relative errors of phase estimation for k = 0 … 5: the whole frequency range 0 – 100 kHz (a) and a zoom between 50 and 80 kHz (b).

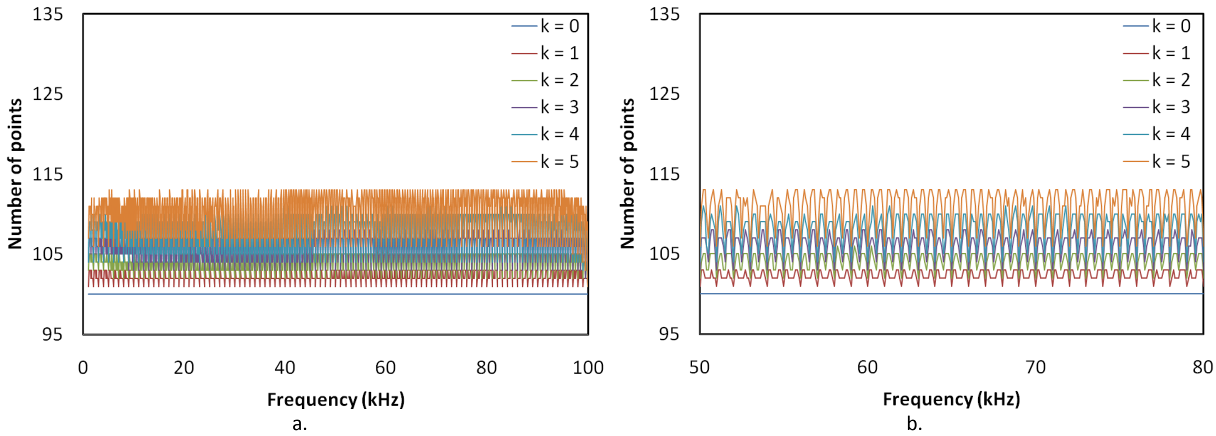

Figure 7. Number of points calculated for k = 0 … 5: the whole frequency range 0 – 100 kHz (a) and a zoom between 50 and 80 kHz (b).

REFERENCES

[1] R.C. Maher, J.W. Beauchamp, Fundamental frequency estimation of musical signals using a two-way mismatch procedure, Journal of the Acoustical Society of America, vol. 95, no. 4, 1994.

[2] M. Christensen, S. Jensen, On perceptual distortion minimization and nonlinear least-squares frequency estimation, IEEE Transactions on Audio, Speech and Language Processing, vol. 14, no. 1, 2006.

[3] Z.N. Milivojevic, M.D. Mirkovic, Estimation of the fundamental frequency of the speech signal modeled by the SYMPES method, International Journal of Electronics and Communications, vol. 63, no. 3, 2009.

[4] G. Seshadri, B. Yegnanarayana, Performance of an Event-Based Instantaneous Fundamental Frequency Estimator for Distant Speech Signals, IEEE Transactions on Audio, Speech and Language Processing, vol. 19, no. 7, 2011.

[5] C. Teague, Root-MUSIC direction finding applied to multifrequency coastal radar, IEEE International Geoscience and Remote Sensing Symposium, vol. 3, 2002.

[6] J. Gai, F. Chan, Y. T. Chan, H. J. Du, F. A. Dilkes, Frequency Estimation of Uncooperative Coherent Pulse Radars, IEEE Military Communications Conference 2007.

[7] J. W. Cooley, J. W. Tukey, An algorithm for the machine calculation of complex Fourier series, Math. Comput., vol. 19, pp. 297–301, 1965.

[8] A. Testa, D. Gallo, R. Langella, "On the processing of harmonics and interharmonics: Using Hanning window in standard framework," IEEE Transactions on Power Delivery, vol. 19, no. 1, pp. 28-34, Jan., 2004.

[9] J. Barros, R.I. Diego, "On the use of the Hanning window for harmonic analysis in the standard framework," IEEE Transactions on Power Delivery, vol. 21, no.1, pp. 28–34, Jan., 2006.

[10] H.A. Gaberson, A comprehensive windows tutorial, Sound and Vibration, vol. 40, no. 3, pp. 14-23, March, 2006.

[11] I.S. Rejin, B.D. Reljin, V.D. Papic, Extremely flat-top windows for harmonic analysis, IEEE Transaction on Instrument and Measurement, vol. 56, no.3, pp. 1025-1041, June, 2007.

[12] C.A. Cortés, E. Mombello, R. Dib, A new class of flat-top windows for exposure assessment in magnetic field measurements, Signal Processing, vol. 87, no. 9, pp. 2151–2164, Sept., 2007.

[13] F. J. Harris, On the use of Windows for Harmonic Analysis with the Discrete Fourier Transform, Proc. IEEE, vol. 66, no. 1, pp. 51-83, Jan., 1978.

[14] M. Novotny, M. Sedlacek, The influence of sidelobes on DFT-based multifrequency signal measurement, *Computer Standard & Interfaces*, vol. 32, no. 3, pp. 110-118, March, 2010.

[15] Y.V. Zakharov, V.M. Baronkin, T.C. Tozer, DFT-based frequency estimators with narrow acquisition range, IEE Proceedings Communications, vol. 148, no. 1, 2001.

[16] B.G. Quinn, Estimating frequency by interpolation using Fourier coefficients, IEEE Transactions on Signal Processing, vol. 42, no. 5, 1994.

[17] B.G. Quinn, Estimation of frequency, amplitude and phase from the DFT of a time series, IEEE Transactions on Signal Processing, vol. 45, no. 3, 1997.

[18] M.D. Macleod, Fast nearly ML estimation of the parameters of real or complex single tones or resolved multiple tones, IEEE Transactions on Signal Processing, vol. 46, no. 1, 1998.

[19] S. Reisenfeld and E. Aboutanios, A new algorithm for the estimation of the frequency of a complex exponential in additive Gaussian noise, IEEE Communication Letters, vol. 7, no. 11, 2003.