Fundamental Frequency Estimation Based on Mean Values

Andrei Sebastian Ardeleanu, Marinel Temneanu
"Gheorghe Asachi" Technical University of Iasi, Faculty of Electrical Engineering, 21-23 Profesor Dimitrie Mangeron Blvd, 700050, Iasi, Romania, a.ardeleanu@ee.tuiasi.ro, mtemnean@ee.tuiasi.ro

***Abstract*-Frequency estimation is an important subject of research in different research areas, such as wireless communication, acoustic and speech processing, sonar and radar measurements. Currently there are available methods that use time domain and frequency domain analysis to detect the frequency of a signal. Within this paper a new time domain method is presented, which analyses the mean values of data segments of the investigated signal. The algorithm doesn't require complex operations and gives good results with only two periods of the investigated signal.**

Keywords: frequency estimation, mean values, fundamental frequency

I. Introduction

A periodic signal with a period of T and frequency $f = 1/T$ can be composed of a series of signals whose frequencies are integer multiples of the f frequency: $1f$, $2f$, $3f$, etc. These frequencies are called harmonics. The first harmonic, also called fundamental frequency, is f, the second is $2f$ and so on.

Fundamental frequency estimation for a complex signal is a difficult problem. The classic method of using the Fourier Transform to obtain the signal's frequency spectrum doesn't offer any information regarding the fundamental's frequency, if its amplitude is zero. This situation is found at signals of limited band used in data transmissions, speech recognition, etc. Also, measurement equipments with functions of calculating the fundamental's and higher harmonics frequencies give false results of the fundamental's frequency and implicitly of the whole harmonic spectrum, for different allures of the investigated signal.

According with the definition of harmonics, the fundamental has a frequency equal with the greatest common divisor of the spectral components frequencies. Regardless of which method is used, determining the frequencies of these components is done with a certain error. This error can lead to erroneous results when calculating the greatest common division, because this algorithm represents a divergent method. For example a generated signal can be considered with two spectral components, one of 100 Hz and the other one of 150 Hz. After the signal is analyzed and the spectral components are calculated, let's suppose the values 101 Hz and 151 Hz are obtained. Calculating the greatest common divisor won't lead on obtaining a value in the vicinity of the real frequency of the fundamental of 50 Hz, but in the vicinity of an incorrect value, 1 Hz.

Fundamental frequency estimation is performed in signal processing, with utility in various applications such as wireless communications, acoustic and speech processing [1-4], sonar and radar measurements [5-6], power quality measurements [7], applications based on power line communication [8-9], or other signal processing applications [10-13]. Over time, various methods of frequency estimation were proposed in order to improve the accuracy and time of execution. These can be classified in methods performed in time domain and methods performed in frequency domain. Time domain methods give good results for fundamental frequency estimation, but when an estimation of harmonic frequencies is desired, frequency domain methods are more adequate to use.

Time domain methods use the repetitive character of the measured signals and analyze different features such as peaks, zero-crossings, threshold crossings or other relevant features. Once these features are identified, the time between two correspondent features is measured in order to determine the period of the signal which will then be used to estimate the fundamental frequency. A period of a signal can be correlated with the next one, so autocorrelation methods [14-16] were implemented to detect the signal's period. Linear prediction estimators [17-18] or combinations of linear prediction and autocorrelation estimators [19-21] were also used to detect the fundamental frequency.

Frequency domain methods use the fact that periodic signals present harmonic components which can be identified as peaks placed at equal distances in the magnitude spectrum. The fundamental frequency is estimated according to the location of the highest value of the spectrum obtained with the help of the Discrete Fourier Transform (DFT). The resolution in frequency (Δf) of the DFT depends on the sampling frequency (f_s) and the number of samples (N) taken into consideration:

$$\Delta f = \frac{f_s}{N}. \tag{1}$$

Because of this, the maximum of the DFT can be obtained for a frequency that doesn't correspond to the real frequency of the signal. The real frequency will be between two DFT values and its estimation will be influenced by an error (ε_f) whose value satisfies:

$$-\frac{f_s}{2N} \le \varepsilon_f \le \frac{f_s}{2N}. \quad (2)$$

To reduce this estimation error, interpolation methods in combination with DFT were used. For the two values, between which the real fundamental frequency is present, an interpolation function is applied and the maximum of the interpolated spectrum will correspond to the fundamental frequency. Interpolation using the parametric cubic convolution [3], parabolic interpolation [21] and other interpolation algorithms [22-25] were also used. A different approach to reduce the estimation error used a variable resolution concept, by using a function that in the vicinity of spectral component increases the resolution, while in rest maintains it to a constant value [26].

With similar performances as the DFT algorithms, a method based on Phase Vocoder [27-28] technique was implemented mainly for the audio and speech processing applications. Considering that the frequency is constant in time, this method estimates the frequency based on the difference of the phase $\Delta\phi$ of two successive windows of the signal with a hop size of H samples:

$$\omega = \frac{1}{2\pi H}\Delta\phi. \quad (3)$$

The phases can be calculated using the DFT or a band-pass FIR filter tuned at or near ω, to isolate the component.

When the investigated signal is composed of a small number of sinusoids the maximum likelihood method [29] is often used to estimate the frequency. This method is efficient, with respect to the Cramer-Rao lower bound, but is computationally demanding.

Other methods for fundamental frequency estimation use iterative filtering [30] or numerical differentiation and central Lagrange interpolation with multi-points [31].

In this paper a new algorithm for estimating the fundamental frequency of a signal based on the mean values is proposed. It is a simple algorithm that doesn't requires many resources and gives precise results with only two cycles from the investigated signal. Also the algorithm isn't influenced by the issue discussed earlier in the paper when calculating the greatest common divisor.

II. ALGORITHM DESCRIPTION

Depending on the time evolution, signals can be classified in periodic and non-periodic signals. A signal is periodic if it has a pattern over a certain window of time, called period, which is repeating at regular intervals of time. Non-periodic signals instead, don't satisfy the above mentioned condition.

The period of a signal for which the frequency is estimated can be divided into two semi-periods, one with positive values and the other with negative values. For an ideal signal these two semi-periods are symmetrical so they cancel together and the result of their addition will be zero. Statistically this can be represented by the mean value which is zero for a period of the signal. Using this idea, the period of an ideal signal can be determined as the minimum segment of samples, whose mean value is zero. Knowing the sampling frequency, signal's fundamental frequency is calculated with:

$$f = \frac{f_s}{N_{per}} \quad (4)$$

where f_s is the sampling frequency and N_{per} is the number of samples per period.

For a measured signal, searching the zero-mean value will not always give a correct result because of the d.c. component of the signal which influences the result of the mean value.

Considering the investigated periodic signal $\boldsymbol{s}$ of N samples has more than two periods, $N/2$ segments of $N/2$ samples can be extracted:

$$\boldsymbol{s_i} = \begin{bmatrix} s_i & s_{i+1} & \dots & s_j \end{bmatrix} \quad (5)$$

where $i = 0 \dots \frac{N}{2} - 1$ and $j = i + \frac{N}{2} - 1$.

In Fig. 1a, two and a half periods of a signal, represented by 50 samples, can be observed. For the signal taken into consideration, 25 segments of 25 samples can be subtracted and three of them were represented for exemplification in Fig. 1b-d, namely $\boldsymbol{s_4}$, $\boldsymbol{s_8}$ and $\boldsymbol{s_{12}}$.

Next, $N/2$ mean vectors ($\boldsymbol{m_i}$) are calculated using the previous extracted segments:

$$\boldsymbol{m_i} = \begin{bmatrix} m_{i,0} & \dots & m_{i,k} & \dots & m_{i,K} \end{bmatrix} \quad (6)$$

where $m_{i,k} = \sum_{l=0}^{k} \frac{s_i[l]}{l+1}$, $k = 0 \dots K$ and $K = \frac{N}{2} - 1$.

For the three segments taken into consideration in Fig. 1, the corresponding mean vectors were computed and represented graphically in Fig. 1f-h. The first value is equal with the first sample of the segment, the second value is the mean of the first and second samples, the third value is the mean of the first, second and third samples and so on until the mean values are calculated for all the samples.

Since the signal is periodic, the mean values obtained for a number of samples that represent an integer number of periods will be the same or almost the same for all the mean vectors. This means that the mean vectors, represented graphically, will intersect at points corresponding to the period of the analyzed signal. Knowing the sampling frequency and the intersection points, the frequency can be estimated with (4).

The period of the signal given in Fig. 1a contains 20 samples. When the three mean vectors are represented in the same graph, it can be seen that they intersect at the point where the mean value was calculated for one period of the signal (Fig. 1e – index 19 on the samples axis, since the counting starts from 0).

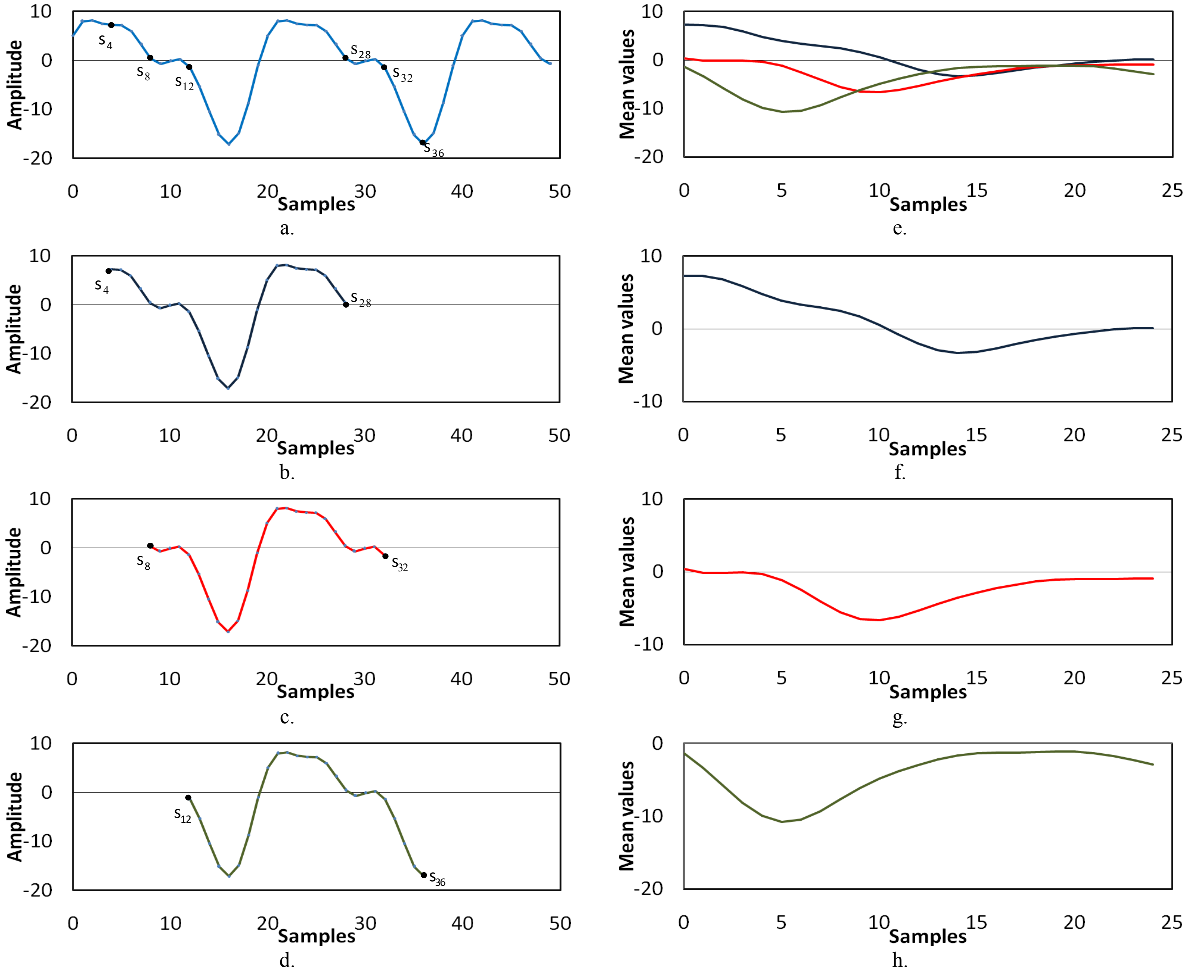

Fig. 1. Signal segmentation

A is a measured signal of 50 samples, B is segment $\boldsymbol{s_4}$ (contains samples from s_4 to s_{28}), C is segment $\boldsymbol{s_8}$ (contains samples from s_8 to s_{32}), D is segment $\boldsymbol{s_{12}}$ (contains samples from s_{12} to s_{36}), E is the graphical representation of the three mean vectors coresponding to segments $\boldsymbol{s_4}$, $\boldsymbol{s_8}$ and $\boldsymbol{s_{12}}$, F is the mean vector of segment $\boldsymbol{s_4}$, G is the mean vector of segment $\boldsymbol{s_8}$ and H is the mean vector of segment $\boldsymbol{s_{12}}$

Also, because of the d.c. component, the mean value obtained at the intersection (corresponding to one period of the signal) is different from zero. Although with only three mean vectors the correct value of the proposed signal's period was identified, for different signals influenced by noise, a larger number of mean vectors is recommended. For the signal given as an example in Fig. 1a, all the 25 mean vectors are represented in Fig. 2. Also in this representation it can be seen that all the vectors intersect at the same point which indicates the signal's period.

The next step in frequency estimation is to determine the point of the intersection. For all the mean vectors the differences between the maxima and minima determined for every index of the vectors are computed. At the point where the mean vectors are intersecting the difference between the maximum and minimum of all vectors has a minimum value. Considering this, with these differences vector $\boldsymbol{d}$ is generated:

$$\boldsymbol{d} = \left[d_0 \quad \dots \quad d_k \quad \dots \quad d_K \right] \tag{7}$$

where $d_k = \max_i m_{i,k} - \min_i m_{i,k}$.

With the help of a search function, the local minima will be found from $\boldsymbol{d}$ vector which will be used to estimate the signal's period and frequency.

III. Conclusions

Frequency estimation methods can be classified in time domain and frequency domain methods. Within this paper, a simple and effective new fundamental frequency estimation algorithm is presented. The algorithm uses a time domain processing which consists in analyzing mean values of data segments to determine the fundamental frequency.

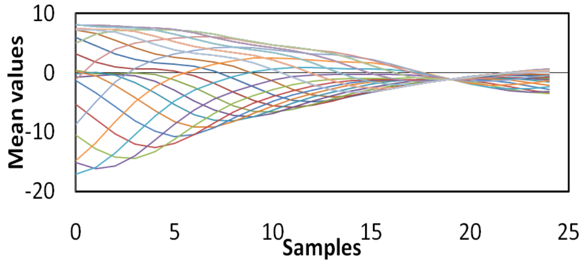

Fig. 2. Graphic representation of 25 mean vectors

The algorithm uses simple operations compared with the ones used in frequency domain methods or within the other time domain methods. It gives good estimations of the fundamental frequency with only two cycles of data from the investigated signal.

Nowadays most of the electrical devices are nonlinear, which results in a current consumption rich in harmonics. The advantages of these devices are: the energy is used more efficiently, the size is considerably reduced together with the costs of production. Their main disadvantage is represented by the pollution of the power supply network with higher harmonics which can interfere with the normal operation of other appliances. Monitoring these harmonics is therefore an important subject of research for power quality monitoring. Analyzing the magnitude of the disturbances introduced by these harmonics can be achieved by extracting the fundamental component from the analyzed signal. In order to achieve this is important to know the fundamental's frequency, which can be determined through the method presented in this paper. Other possible applications of the proposed method are in the field of acoustic and speech processing, sonar and radar measurements, power line communication or other applications that require the analysis of the fundamental frequency of a signal.

ACKNOWLEDGMENT

This work was supported by a grant of the Romanian National Authority for Scientific Research, CNDI–UEFISCDI, project "Contor inteligent bazat pe evaluarea semnaturii energetice" SigMET, 30_PCCA_2012.

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