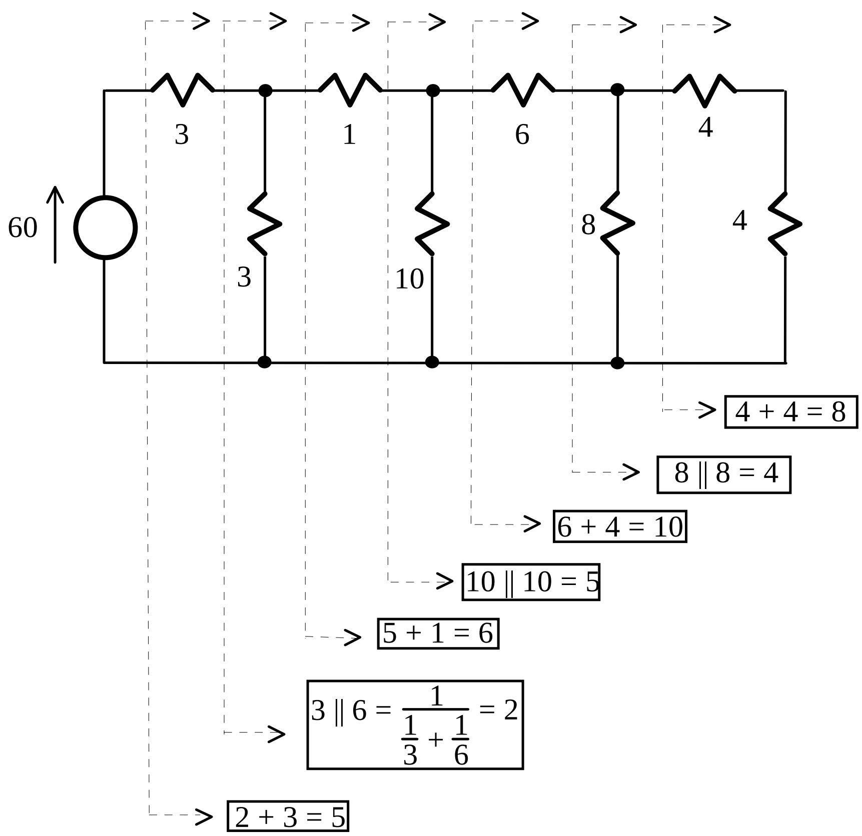
F.3 Analiza Cir El c.c. I

Q.1

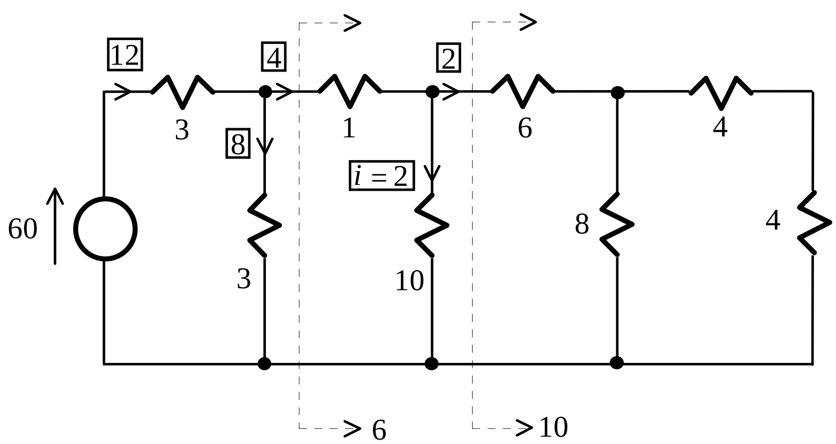
Source current



All units in
V, A, Ω

$$\text{Source current} = \frac{60}{5} = 12\,\text{A}$$

Value for i

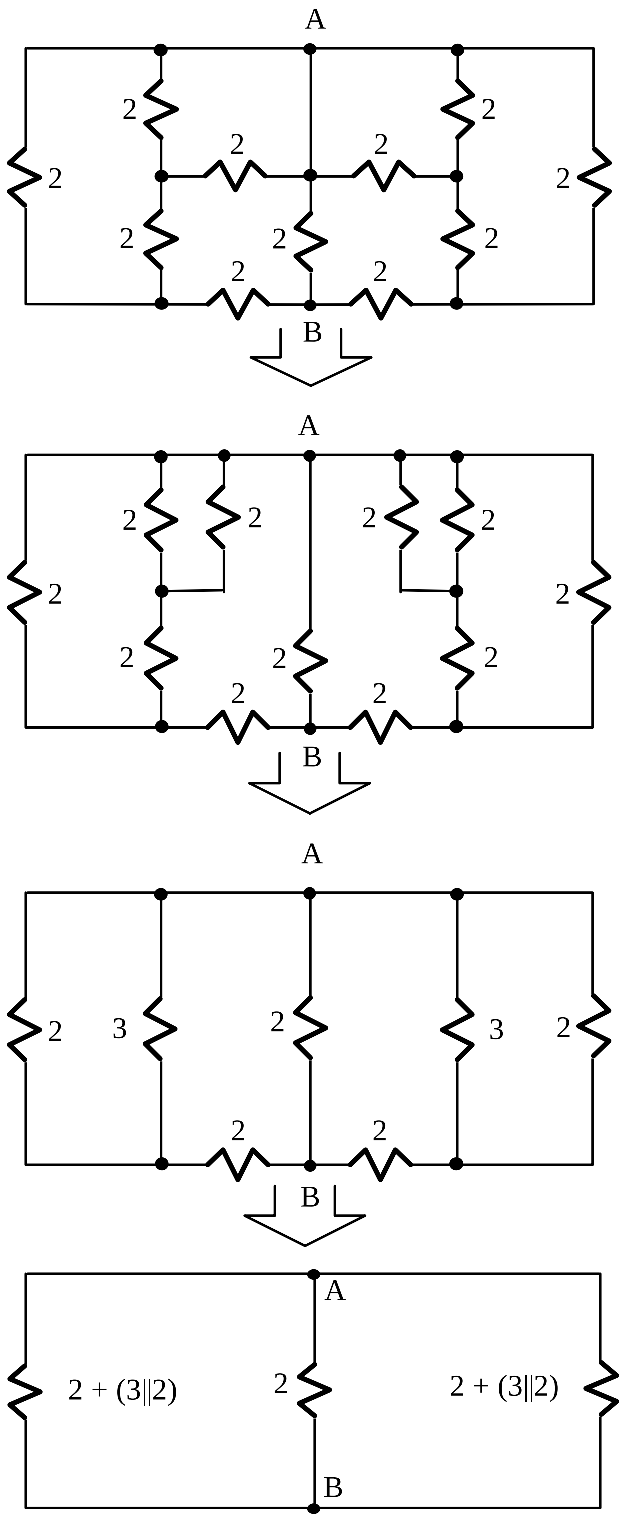


All units in
V, A, Ω

Q.2

Original circuit

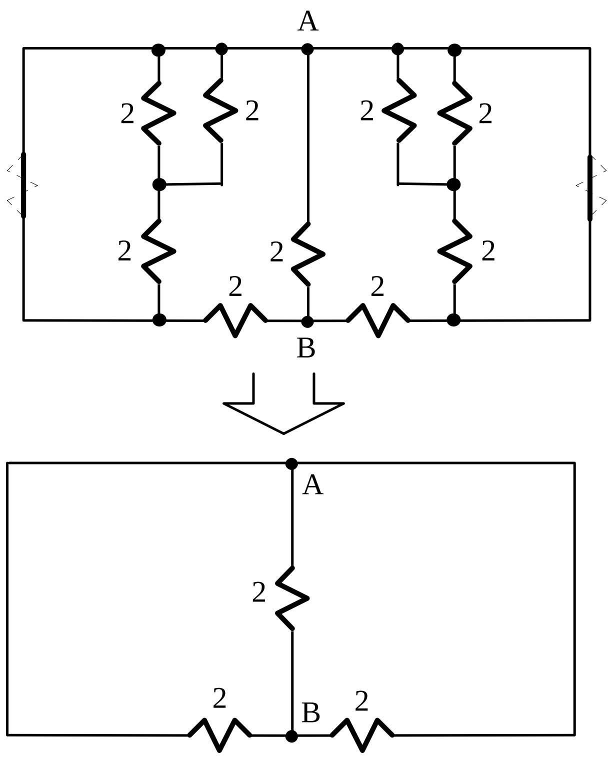
All units in V, A, Ω



$$2 + (3\|2) = 2 + \frac{1}{\frac{1}{3} + \frac{1}{2}} = 2 + \frac{6}{5} = 3.2$$

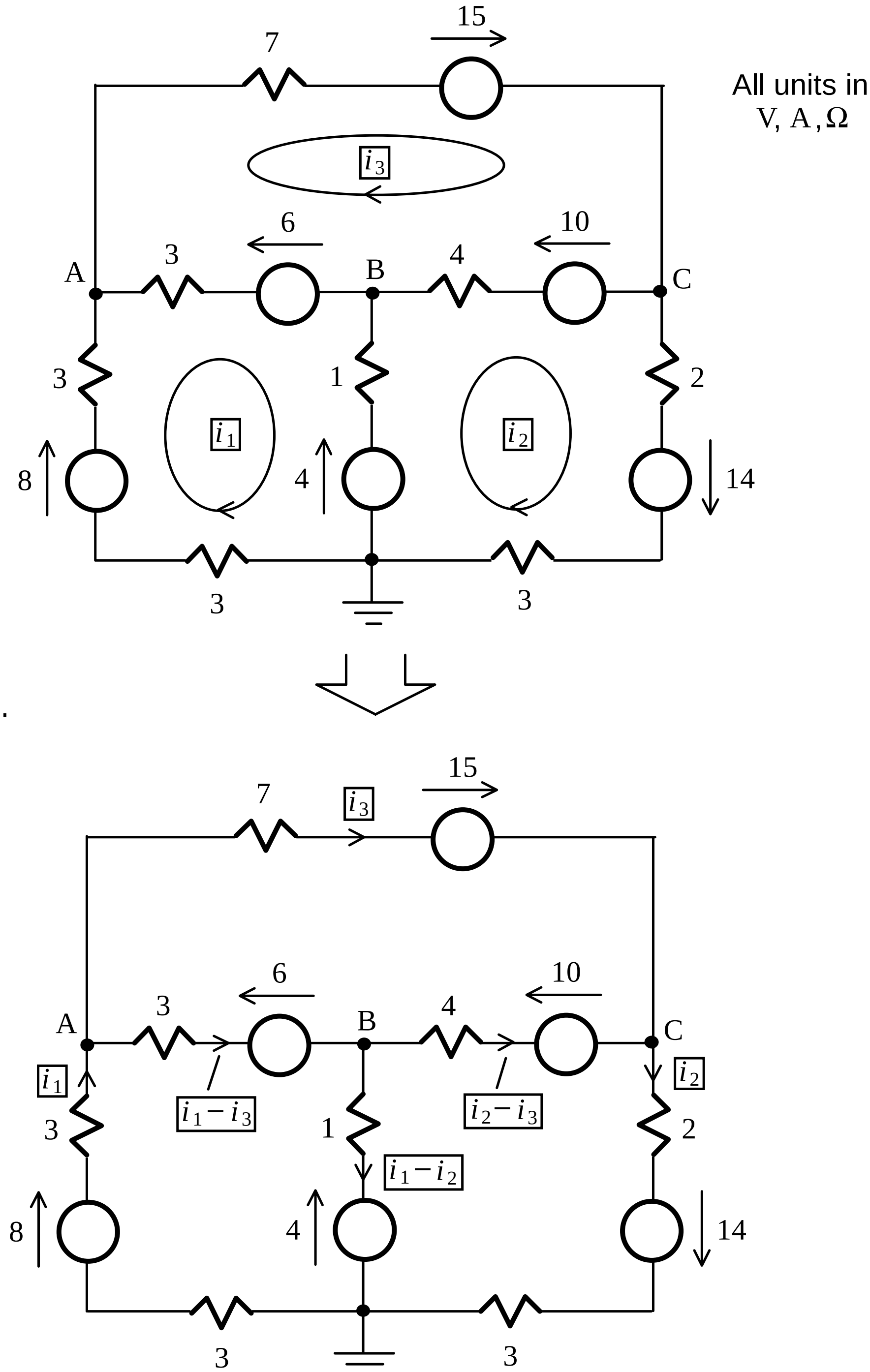
Equivalent resistance $= 2 \,||\, 3.2 \,||\, 3.2 = 2 \,||\, 1.6 = \dfrac{1}{0.5 + 0.625} = 0.889\,\Omega$

When outer two resistors are short-circuited



Equivalent resistance $= 2 \,||\, 2 \,||\, 2 = \dfrac{2}{3} = 0.666\,\Omega$

Q.3 *Mesh analysis*



Applying KVL for the three loops shown:

$$8 - 3i_1 - 3(i_1 - i_3) - 6 + (i_1 - i_2) - 4 - 3i_1 = 0$$

$$14 - 3i_2 + 4 - (i_2 - i_1) - 4(i_2 - i_3) - 10 - 2i_2 = 0$$

$$15 + 10 - 4(i_3 - i_2) + 6 - 3(i_3 - i_1) - 7i_3 = 0$$

Simplifying:

$$-2 = 10i_1 - i_2 - 3i_3$$

$$8 = -i_1 + 10i_2 - 4i_3$$

$$31 = -3i_1 - 4i_2 + 14i_3$$

In matrix form:

$$\begin{bmatrix} 10 & -1 & -3 \\ -1 & 10 & -4 \\ -3 & -4 & 14 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \\ 31 \end{bmatrix}$$

Solving (actually not required in this question):

$$-2+10(8)=10i_1-i_2-3i_3+10(-i_1+10i_2-4i_3) \quad \Rightarrow \quad 78=99i_2-43i_3$$

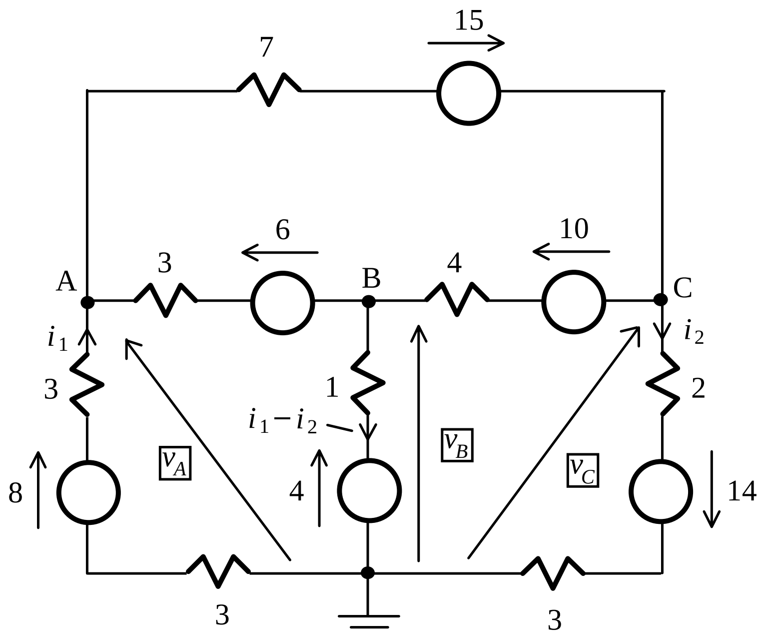
$$31-3(8)=-3i_1-4i_2+14i_3-3(-i_1+10i_2-4i_3) \;\Rightarrow\; 7=-34i_2+26i_3$$

$$26(78)+43(7)=26(99i_2-43i_3)+43(-34i_2+26i_3) \;\Rightarrow\; i_2=\frac{26(78)+43(7)}{26(99)-43(34)}=\frac{2329}{1112}$$

$$34(78)+99(7)=34(99i_2-43i_3)+99(-34i_2+26i_3) \;\Rightarrow\; i_3=\frac{34(78)+99(7)}{-34(43)+99(26)}=\frac{3345}{1112}$$

$$i_1=-8+10i_2-4i_3=-8+\frac{23290}{1112}-\frac{4(3345)}{1112}=\frac{1014}{1112}$$

Voltages of nodes A, B and C with respect to ground:

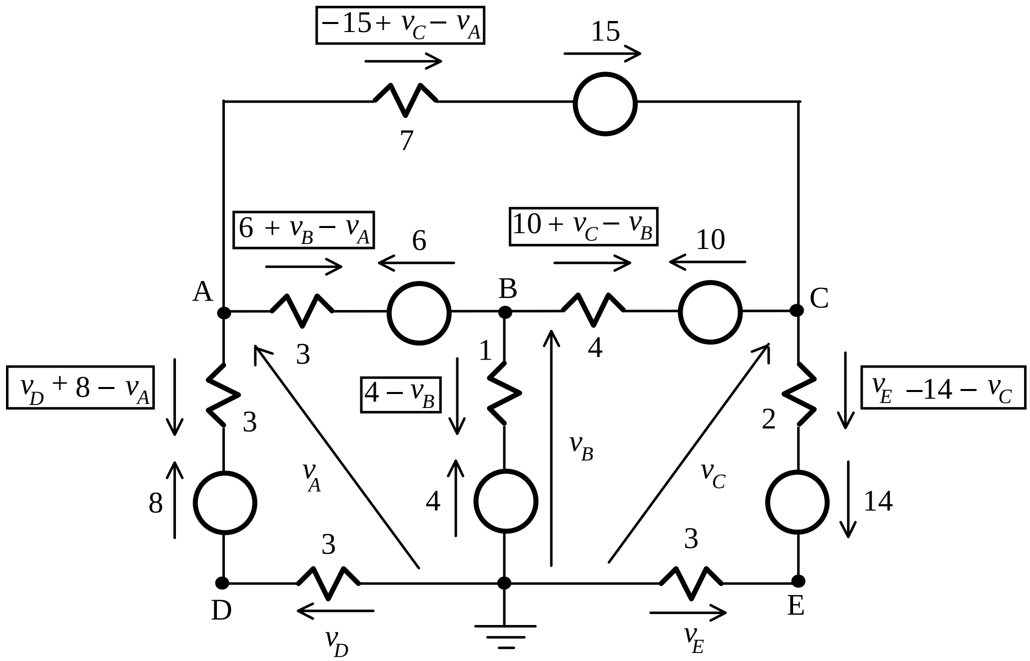


$$v_A=8-6i_1$$

$$v_B=4+i_1-i_2$$

$$v_C=-14+5i_2$$

Nodal analysis



Applying KCL to nodes A, B, C, D and E:

$$\frac{v_D+8-v_A}{3}+\frac{6+v_B-v_A}{3}+\frac{v_C-v_A-15}{7}=0$$

$$\frac{4-v_B}{1}+\frac{v_A-v_B-6}{3}+\frac{v_C-v_B+10}{4}=0$$

$$\frac{v_E-14-v_C}{2}+\frac{v_A-v_C+15}{7}+\frac{v_B-v_C-10}{4}=0$$

$$\frac{v_D+8-v_A}{3}+\frac{v_D}{3}=0$$

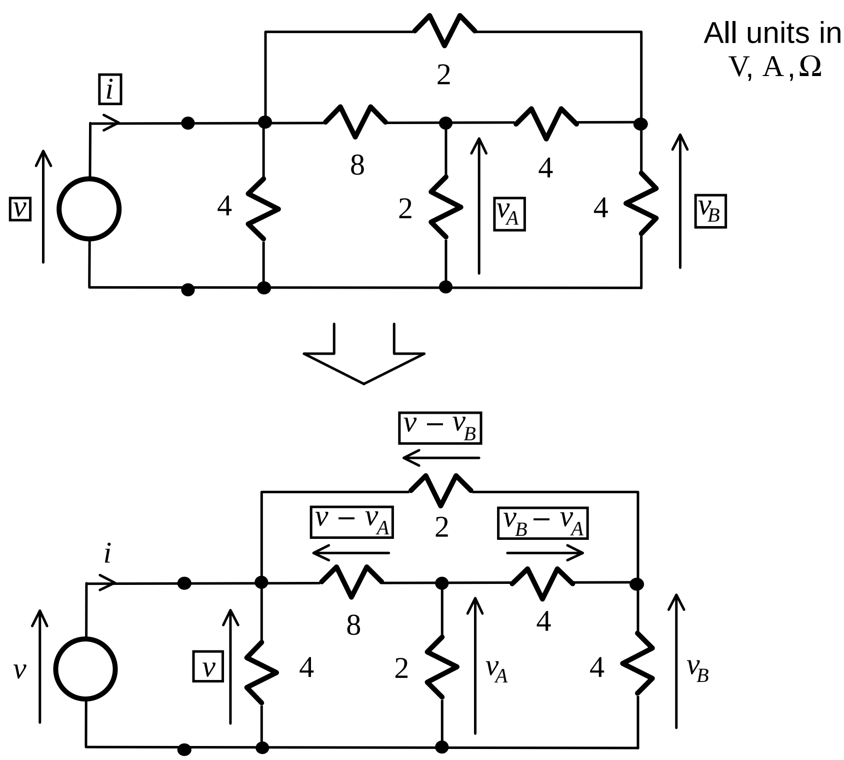
$$\frac{v_E-14-v_C}{2}+\frac{v_E}{3}=0$$

In matrix form:

$$\begin{bmatrix} 1/3+1/3+1/7 & -1/3 & -1/7 & -1/7 & 0 \\ -1/3 & 1+1/3+1/4 & -1/4 & 0 & 0 \\ -1/7 & -1/4 & 1/2+1/7+1/4 & 0 & -1/2 \\ \text{Á}/3 & 0 & 0 & -1/3-1/3 & 0 \\ 0 \quad 0 \quad 1/2 \quad 0 \quad -1/2-1/3 & & & & \end{bmatrix}\begin{bmatrix} v_A \\ v_B \\ v_C \\ v_D \\ v_E \end{bmatrix}=\begin{bmatrix} 8/3+6/3-15/7 \\ 1/4-2+10/4 \\ -7+15/7-10/4 \\ 8/3 \\ -14/2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{3}+\frac{1}{3}+\frac{1}{7} & -\frac{1}{3} & & & \\ & 1+\frac{1}{3}+\frac{1}{4} & & & \\ & & \frac{1}{2}+\frac{1}{7}+\frac{1}{4} & & \\ & & & -\frac{1}{3}-\frac{1}{3} & \\ & & & & -\frac{1}{2}-\frac{1}{3} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \\ v_D \\ v_E \end{bmatrix} = \begin{bmatrix} \frac{8}{3}+\frac{6}{3}-\frac{15}{7} \\ \frac{1}{4}-2+\frac{10}{4} \\ -7+\frac{15}{7}-\frac{10}{4} \\ \frac{8}{3} \\ -\frac{14}{2} \end{bmatrix}$$

Q.4



Applying KCL:

$$i = \frac{v}{4} + \frac{v - v_A}{8} + \frac{v - v_B}{2}$$

$$\frac{v_A}{2} + \frac{v_A - v}{8} + \frac{v_A - v_B}{4} = 0 \;\Rightarrow\; 4v_A + v_A - v + 2v_A - 2v_B = 7v_A - 2v_B - v = 0$$

$$\frac{v_B}{4} + \frac{v_B - v_A}{4} + \frac{v_B - v}{2} = 0 \;\Rightarrow\; v_B + v_B - v_A + 2v_B - 2v = -v_A + 4v_B - 2v = 0$$

Eliminating v_A and v_B:

$$(7v_A - 2v_B - v) + 7(-v_A + 4v_B - 2v) = 0 \;\Rightarrow\; 26v_B - 15v = 0 \;\Rightarrow\; v_B = \frac{15}{26}v$$

$$2(7v_A - 2v_B - v) + (-v_A + 4v_B - 2v) = 0 \;\Rightarrow\; 13v_A - 4v = 0 \;\Rightarrow\; v_A = \frac{4}{13}v$$

$$i = \frac{v}{4} + \frac{v - v_A}{8} + \frac{v - v_B}{2} = \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{2}\right)v - \frac{1}{8}\left(\frac{4}{13}\right)v - \frac{1}{2}\left(\frac{15}{26}\right)v = \frac{57}{104}v$$

The equivalent resistance without the 13Ω resistor is therefore

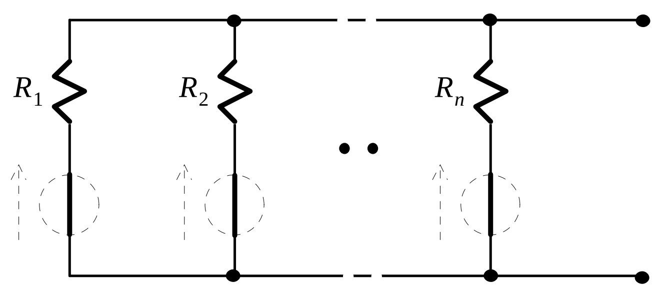
$$\frac{v}{i} = \frac{104}{57} = 1.82\,\Omega$$

and the equivalent resistance with the 13Ω series resistor is

Equivalent resistance $= 14.82\,\Omega$

Q.5

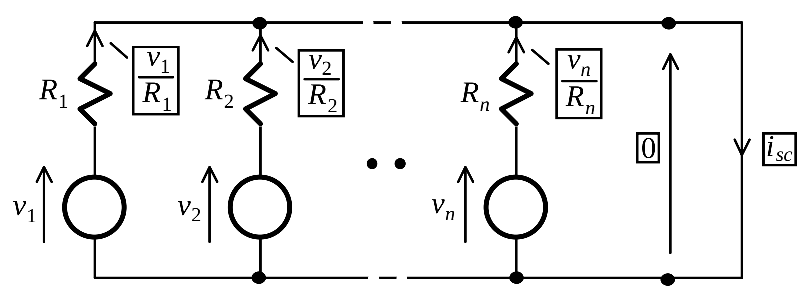
Equivalent resistance



Since the resistors are in parallel, the equivalent resistance R is

$$\frac{1}{R} = \frac{1}{R_1} + \cdots + \frac{1}{R_n}$$

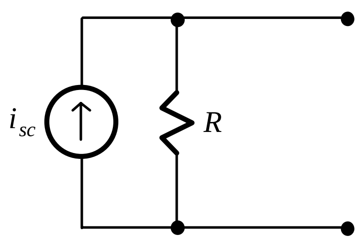
Short circuit current



Applying KCL:

$$i_{sc} = \frac{v_1}{R_1} + \cdots + \frac{v_n}{R_n}$$

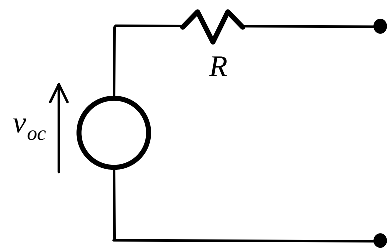
Norton's equivalent circuit



$$\frac{1}{R}=\frac{1}{R_1}+\cdots+\frac{1}{R_n}$$

$$i_{sc}=\frac{v_1}{R_1}+\cdots+\frac{v_n}{R_n}$$

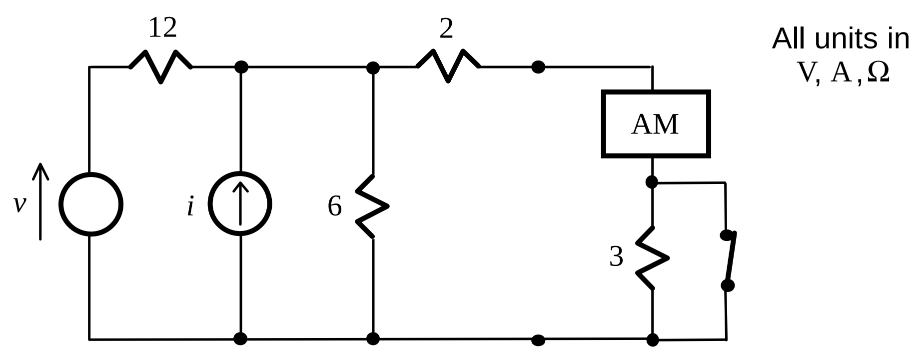
Thevenin's equivalent circuit



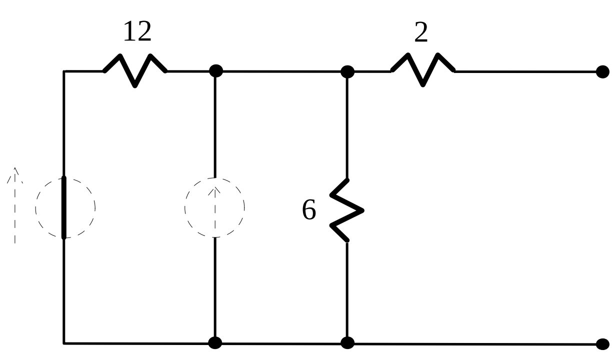
$$\frac{1}{R}=\frac{1}{R_1}+\cdots+\frac{1}{R_n}$$

$$v_{oc}=Ri_{sc}=\frac{\frac{v_1}{R_1}+\cdots+\frac{v_n}{R_n}}{\frac{1}{R_1}+\cdots+\frac{1}{R_n}}$$

Q.6 *Re-drawing original circuit*

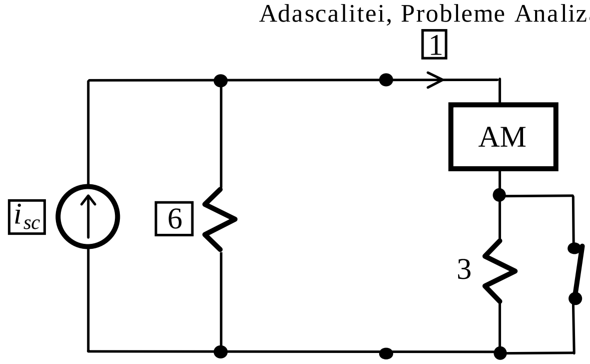


Equivalent resistance without 3Ω *resistor*



Equivalent resistance $R=2+\left(6||12\right)=2+\frac{1}{\frac{1}{6}+\frac{1}{12}}=2+4=6\Omega$

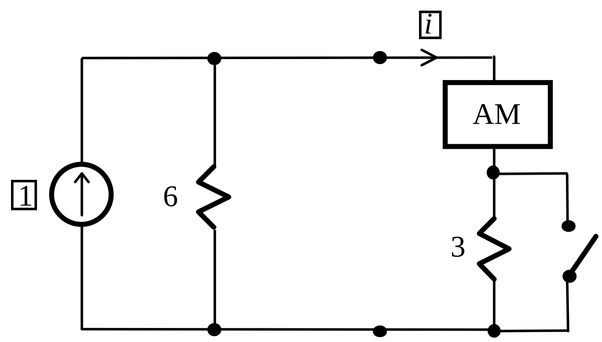
Using Norton's equivalent circuit



Since AM reads 1A

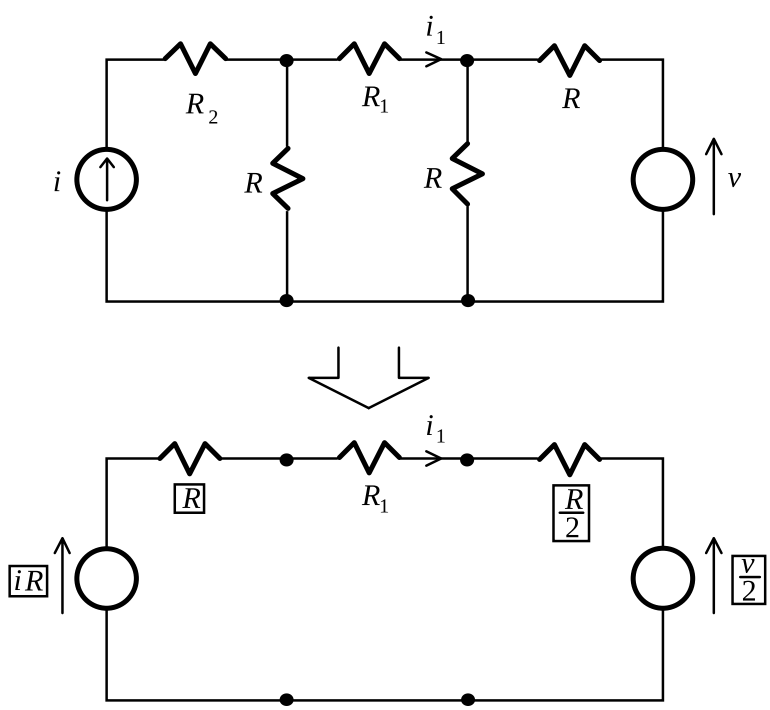
$i_{sc} = 1\text{A}$

Thus, when the switch is open



$$i = 1 \times \left(\frac{6}{6+3} \right) = \frac{2}{3}\text{A}$$

Q.7

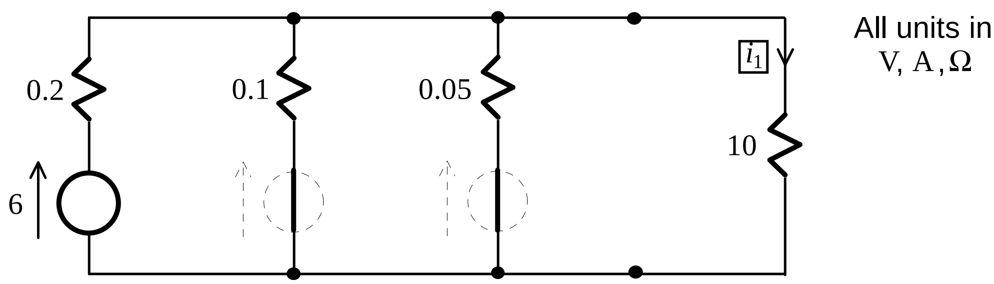


$$i_1 = \frac{iR - \frac{v}{2}}{\frac{3R}{2} + R_1} = \frac{2iR - v}{3R + 2R_1}$$

F.4 Analiza Cir El c.c. II

Q.1

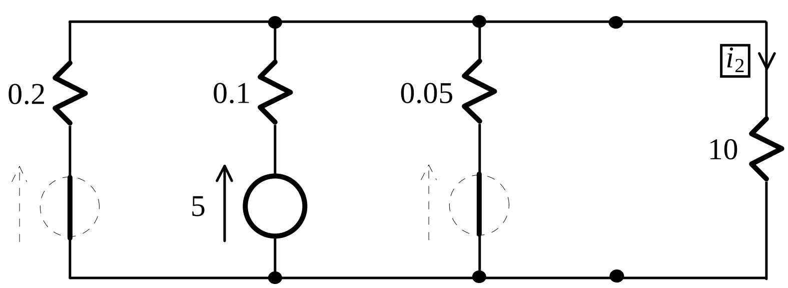
Load current due to Battery 1



$$\text{Current from source} = \frac{6}{0.2 + 0.1||0.05||10} = \frac{6}{0.2 + \frac{1}{10+20+0.1}} = \frac{6}{0.2 + 0.033} = 25.75$$

$$i_1 = 25.75\times\left(\frac{0.1||0.05}{10 + 0.1||0.05}\right) = 25.75\times\left(\frac{\frac{1}{10+20}}{10 + \frac{1}{10+20}}\right) = 25.75\times\left(\frac{0.033}{10.033}\right) = 0.0847\,\text{A}$$

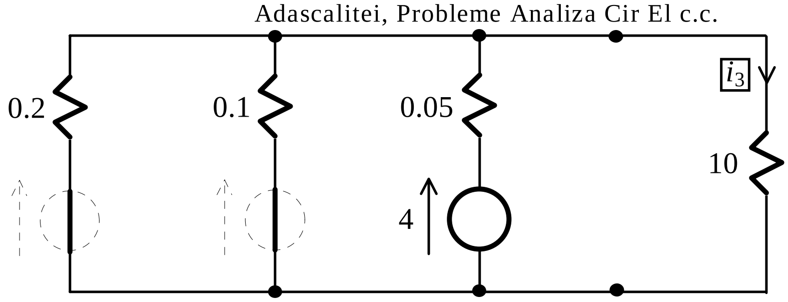
Load current due to Battery 2



$$\text{Current from source} = \frac{5}{0.1 + 0.2||0.05||10} = \frac{5}{0.1 + \frac{1}{5+20+0.1}} = \frac{5}{0.1 + 0.04} = 35.71$$

$$i_2 = 35.71\times\left(\frac{0.2||0.05}{10 + 0.2||0.05}\right) = 35.71\times\left(\frac{\frac{1}{5+20}}{10 + \frac{1}{5+20}}\right) = 35.71\times\left(\frac{0.04}{10.04}\right) = 0.1423\,\text{A}$$

Load current due to Battery 3



$$\text{Current from source} = \frac{4}{0.05+0.2||0.1||10} = \frac{4}{0.05+\frac{1}{5+10+0.1}} = \frac{4}{0.05+0.066} = 34.48$$

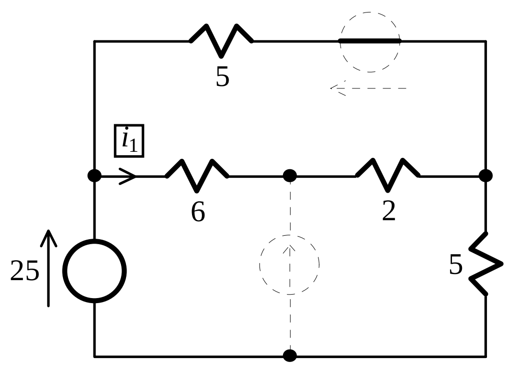
$$i_3 = 34.48\times\left(\frac{0.2||0.1}{10+0.2||0.1}\right) = 34.48\times\left(\frac{\frac{1}{5+10}}{10+\frac{1}{5+10}}\right) = 34.48\times\left(\frac{0.067}{10.07}\right) = 0.2294\,\text{A}$$

Actual load current

$$i = i_1 + i_2 + i_3 = 0.0847 + 0.1423 + 0.2294 = 0.4564\,\text{A}$$

Q.2

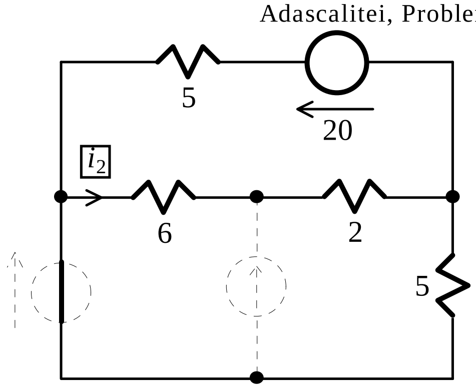
Current due to $25\,\text{V}$ *voltage source*



$$\text{Current from source} = \frac{25}{5+5||(6+2)} = \frac{25}{5+\frac{1}{\frac{1}{5}+\frac{1}{8}}} = \frac{25}{5+3.08} = 3.09$$

$$i_1 = 3.09\left(\frac{5}{5+8}\right) = 1.19\,\text{A}$$

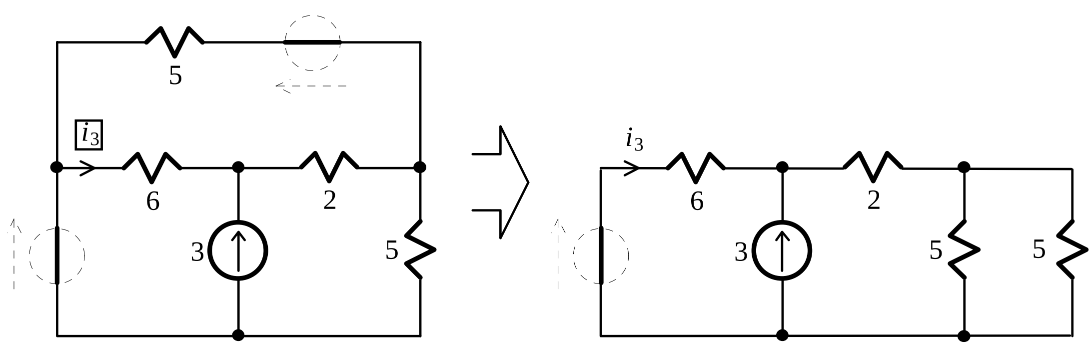
Current due to $20\,\text{V}$ *voltage source*



Current from source $= \frac{20}{5+5\|(6+2)} = \frac{20}{5+\frac{1}{\frac{1}{5}+\frac{1}{8}}} = \frac{20}{5+3.08} = 2.48$

$$i_2 = 2.48\left(\frac{5}{5+8}\right) = 0.954\,\mathrm{A}$$

Current due to current source



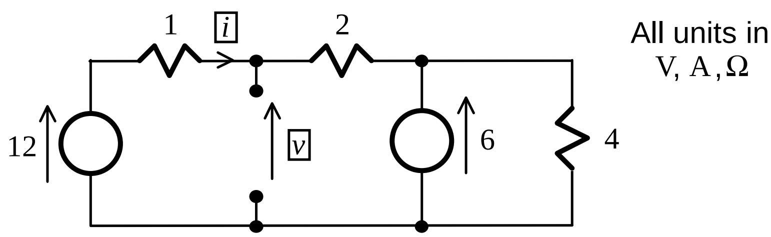
$$i_3 = -3\left[\frac{2+(5\|5)}{6+2+(5\|5)}\right] = -3\left[\frac{2+\frac{1}{0.2+0.2}}{8+\frac{1}{0.2+0.2}}\right] = -1.286\mathrm{A}$$

Actual current

$$i = i_1 + i_2 + i_3 = 1.19 + 0.954 - 1.286 = 0.858\,\mathrm{A}$$

Q.3

Open circuit voltage

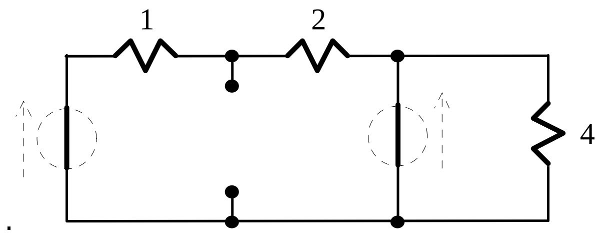


.

$$i = \frac{12-6}{1+2} = 2$$

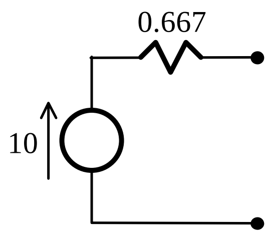
$v = 12 - i = 10\,\mathrm{V}$

Equivalent resistance



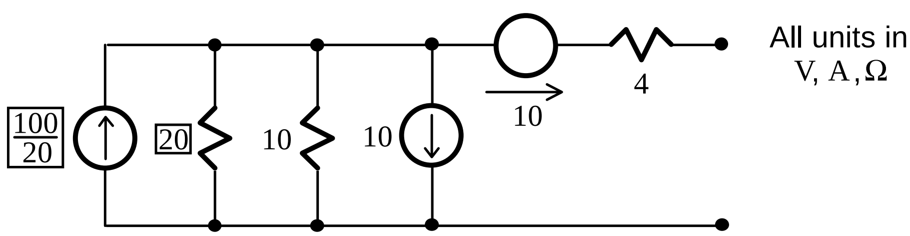
Resistance across terminals $= 1 \| 2 = \dfrac{1}{1 + \frac{1}{2}} = 0.667\,\Omega$

Thevenin's equivalent circuit and maximum power



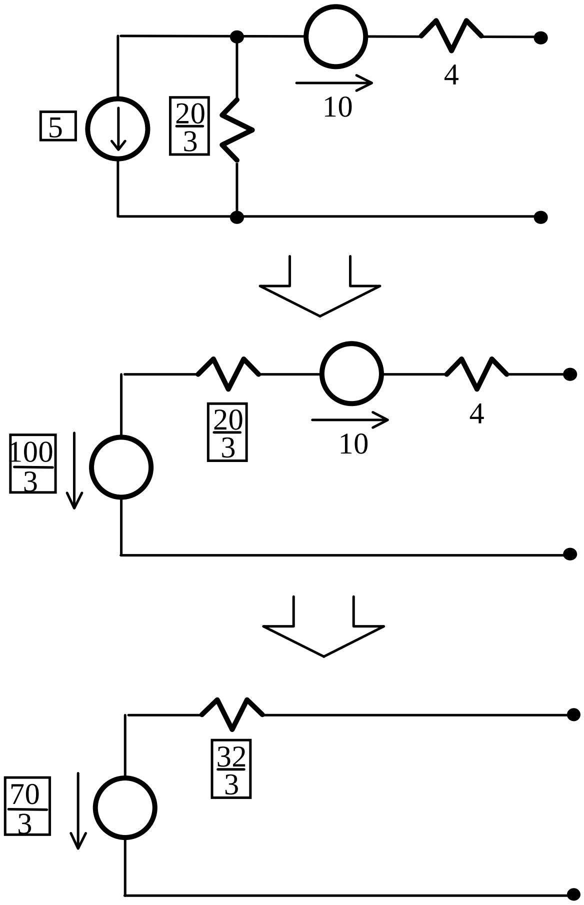
Maximum power transferable (with a $0.667\,\Omega$ load) $= \left(\dfrac{10}{2 \times 0.667}\right)^2 0.667 = 37.5\,\mathrm{W}$

Q.4



Combined current of current sources $= 10 - \dfrac{100}{20} = 5$

Equivalent parallel resistance $= 20 \| 10 = \dfrac{1}{\frac{1}{20} + \frac{1}{10}} = \dfrac{20}{3}$



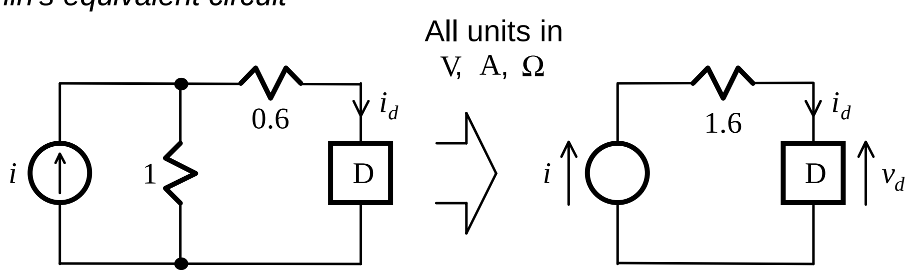
Resistor that draws $2\,\text{A} = \frac{70}{3\times 2} - \frac{32}{3} = 1\Omega$

Resistor that absorbs the maximum power $= \frac{32}{3}\Omega$

Maximum power that can be transferred $= \left(\frac{70}{3} \Big/ \frac{32\times 2}{3}\right)^2 \frac{32}{3} = \frac{1225}{96}\text{W}$

Q.5

Thevenin's equivalent circuit

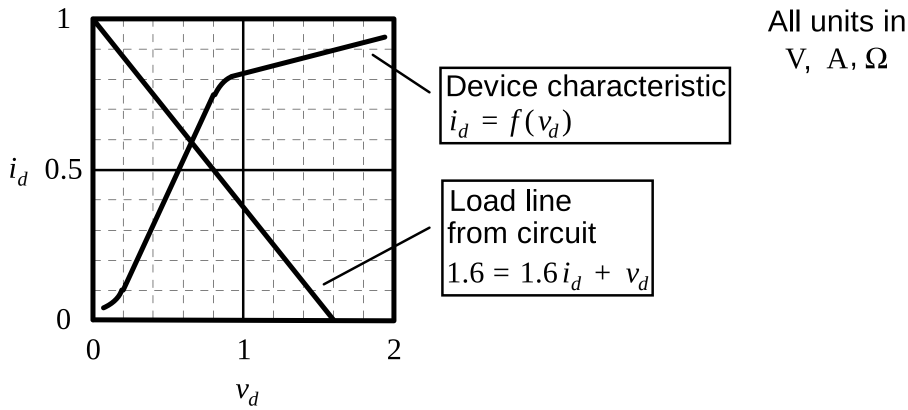


Device current given $i = 1.6\,\text{A}$

Applying KVL:

$i = 1.6 i_d + v_d = 1.6$

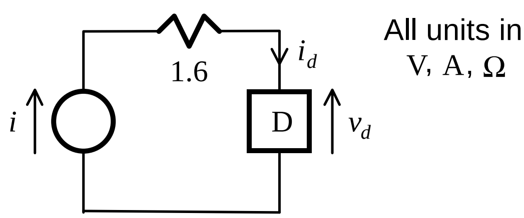
v_d and i_d can be found from solving this (which gives rise to the load line) and the relationship $i_d = f(v_d)$ given by the characteristic curve. Specifically, when $i_d = 0$, $v_d = 1.6$. Also, when $v_d = 0$, $i_d = 1$.



The point of intersection gives

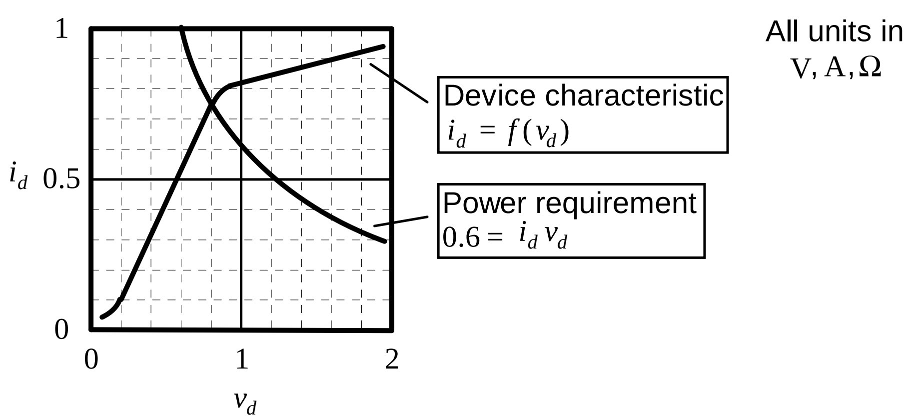
$i_d = 0.6\,\text{A}$

Source current for power dissipated in D *to be* 0.6 W



Power dissipated in $\text{D} = i_d v_d = 0.6$

The device voltage and current can be found from solving this and the relationship $i_d = f(v_d)$ given by the characteristic curve:



The point of intersection gives

$v_d \approx 0.8\,\text{V}$

$i_d \approx 0.75\,\text{A}$

From KVL:

$$i = 1.6 i_d + v_d \approx 1.6(0.75) + 0.8 = 2\,\mathrm{A}$$

Q.6

Voltage gain

Applying KCL to the second half of the circuit:

$$v_2 = -20(50 i_1) = -1000 i_1$$

Applying KVL to the first half of the circuit:

$$v_1 = 2 i_1 + \frac{v_2}{5000}$$

Eliminating i_1:

$$v_1 = -2\left(\frac{v_2}{1000}\right) + \frac{v_2}{5000} = \frac{-1.8 v_2}{1000} \quad \Rightarrow \quad v_2 = -\frac{1000}{1.8} v_1$$

The voltage gain is

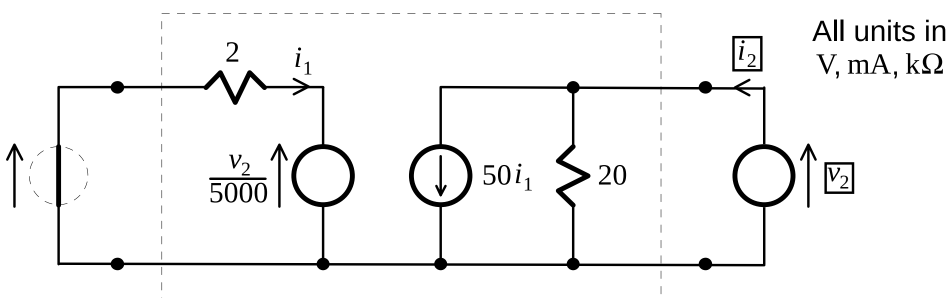
$$\frac{v_2}{v_1} = -\frac{1000}{1.8}$$

The gain in voltage magnitudes is

$$\left|\frac{v_2}{v_1}\right| = \frac{1000}{1.8} = 20\log\left(\frac{1000}{1.8}\right)\mathrm{dB} = 55\mathrm{dB}$$

Equivalent resistance

To determine the equivalent resistance as seen from the output terminals, all independent sources have to be replaced by their internal resistances and a voltage source has to be applied to these two terminals:



Transistor amplifier

$$-2 i_1 = \frac{v_2}{5000}$$

$$i_2 = 50 i_1 + \frac{v_2}{20} = -50\left(\frac{v_2}{10000}\right) + \frac{v_2}{20} = \frac{9 v_2}{200}$$

Equivalent resistance $= \frac{v_2}{i_2} = \frac{200}{9}\,\text{k}\Omega$

Note that in calculating this resistance or in using superposition, dependent sources must not be replaced by their internal resistances.

Thevenin's equivalent circuit