F.5 Analiza Cir El c.a. I

Q.1	(a)	(b)
AC waveform	$5\sqrt{2}\sin(\omega t)$ $=5\sqrt{2}\cos\left(\omega t-\frac{\pi}{2}\right)$ $=\mathrm{Re}\left[\left(5e^{-j\pi/2}\right)\left(\sqrt{2}e^{j\omega t}\right)\right]$	$5\sqrt{2}\cos(\omega t)$ $=\mathrm{Re}\left[\left(5e^{j0}\right)\left(\sqrt{2}e^{j\omega t}\right)\right]$
Peak value	$5\sqrt{2}$	$5\sqrt{2}$
Frequency	$\omega\,\mathrm{rad/s}=\frac{\omega}{2\pi}\,\mathrm{Hz}$	$\omega\,\mathrm{rad/s}=\frac{\omega}{2\pi}\,\mathrm{Hz}$
RMS value	5	5
Phase	$-\frac{\pi}{2}=-90^0$	0
Phasor	$5\ e^{-j\pi/2}=5\ \underline{/-90^o}$	$5\ e^{j0}=5\ \underline{/0^o}=5$

	(c)	(d)
AC waveform	$10\sqrt{2}\sin\left(20t+30^0\right)$ $=10\sqrt{2}\cos\left(20t-60^0\right)$ $=\mathrm{Re}\left[\left(10e^{-j\pi/3}\right)\left(\sqrt{2}e^{j20t}\right)\right]$	$120\sqrt{2}\cos\left(314t-45^0\right)$ $=\mathrm{Re}\left[\left(120e^{-j\pi/4}\right)\left(\sqrt{2}e^{j314t}\right)\right]$
Peak value	$10\sqrt{2}$	$120\sqrt{2}$
Frequency	$20\,\mathrm{rad/s}=3.18\,\mathrm{Hz}$	$314\,\mathrm{rad/s}=50\,\mathrm{Hz}$
RMS value	10	120
Phase	$-\frac{\pi}{3}=-60^0$	$-\frac{\pi}{4}=-45^0$
Phasor	$10\,e^{-j\pi/3}=10\underline{/-60^o}$	$120e^{-j\pi/4}=120\underline{/-45^o}$

	(e)	(f)
AC waveform	$-50\sin\left(4t-\frac{\pi}{3}\right)$ $=35.4\sqrt{2}\cos\left(4t-\frac{\pi}{3}-\frac{\pi}{2}+\pi\right)$ $=\mathrm{Re}\left[\left(35.4e^{j\pi/6}\right)\left(\sqrt{2}e^{j4t}\right)\right]$	$0.25\cos\left(2t+100^0\right)$ $=0.177\sqrt{2}\cos\left(2t+1.75\right)$ $=\mathrm{Re}\left[\left(0.177e^{j1.75}\right)\left(\sqrt{2}e^{j2t}\right)\right]$
Peak value	50	0.25
Frequency	$4\,\mathrm{rad/s}=0.637\,\mathrm{Hz}$	$2\,\mathrm{rad/s}=0.318\,\mathrm{Hz}$
RMS value	35.4	0.177
Phase	$\frac{\pi}{6}=30^0$	$1.75=100^0$
Phasor	$35.4e^{j\pi/6}=35.4\angle 30^o$	$0.177e^{j1.75}=0.177\angle 100^o$

Q.2

(a)	$\frac{100}{\sqrt{2}}e^{j30^0}\ \mathrm{V}$	$\mathrm{Re}\left[\left(\frac{100}{\sqrt{2}}e^{j\pi/6}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]=100\cos\left(314t+\frac{\pi}{6}\right)\mathrm{V}$
(b)	$115e^{j\pi/3}\ \mathrm{V}$	$\mathrm{Re}\left[\left(115e^{j\pi/3}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]=115\sqrt{2}\cos\left(314t+\frac{\pi}{3}\right)\mathrm{V}$
(c)	$-0.12e^{-j\pi/4}\ \mathrm{A}$	$\mathrm{Re}\left[\left(-0.12e^{-j\pi/4}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]=\mathrm{Re}\left[\left(e^{j\pi}0.12e^{-j\pi/4}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]$ $=0.12\sqrt{2}\cos\left(314t+\frac{3\pi}{4}\right)\mathrm{A}$
(d)	$-0.69\angle 60^o\ \mathrm{A}$	$\mathrm{Re}\left[\left(-0.69e^{j\pi/3}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]=\mathrm{Re}\left[\left(e^{j\pi}0.69e^{j\pi/3}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]$ $=0.69\sqrt{2}\cos\left(314t+\frac{4\pi}{3}\right)$ $=0.69\sqrt{2}\cos\left(314t-\frac{2\pi}{3}\right)\mathrm{A}$

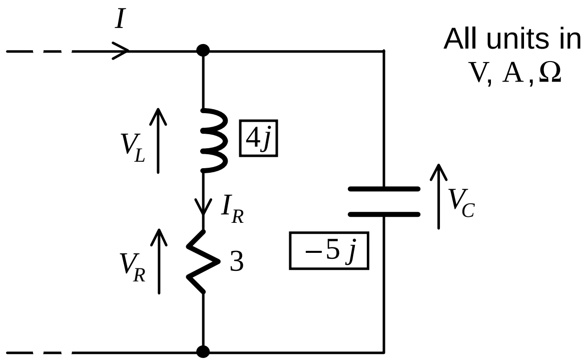
Q.3 From $v_R(t)=12\sqrt{2}\cos\left(2t\right)\mathrm{V}$

Frequency $=\omega=2\,\mathrm{rad/s}$

Impedance of capacitor $= \dfrac{1}{j\omega 0.1} = \dfrac{1}{j0.2} = -5j\,\Omega$

Impedance of inductor $= j\omega 2 = 4j\,\Omega$

$V_R = 12e^{j0} = 12$

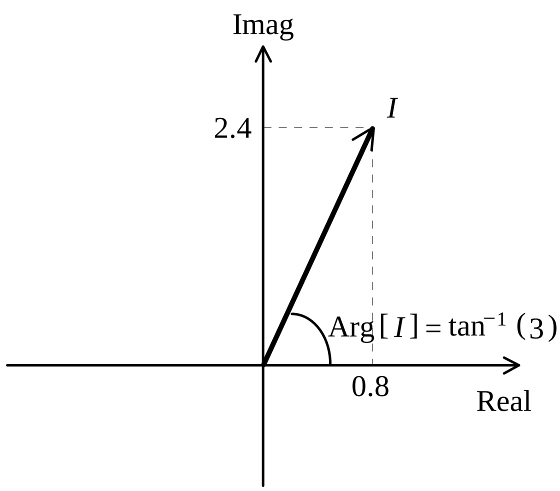


$I_R = \dfrac{V_R}{3} = 4 \;\Rightarrow\; i_R(t) = 4\sqrt{2}\cos(2t)\ \text{A}$

$V_L = (4j)I_R = 16j\ \text{V} \;\Rightarrow\; v_L(t) = 16\sqrt{2}\cos\left(2t + \dfrac{\pi}{2}\right)\text{V}$

$V_C = V_R + V_L = 12 + 16j$

$I = I_R + \dfrac{V_C}{-5j} = 4 - \dfrac{12+16j}{5j} = 4 + 2.4j - 3.2 = 0.8 + 2.4j$



$|I| = |0.8 + 2.4j| = \sqrt{0.8^2 + 2.4^2} = 2.53$

$\text{Arg}[I] = \text{Arg}[0.8 + 2.4j] = \tan^{-1}\left(\dfrac{2.4}{0.8}\right) = 1.25$

$I = 0.8 + 2.4j = 2.53e^{j1.25} \;\Rightarrow\; 2.53\sqrt{2}\cos(2t + 1.25)\ \text{A}$

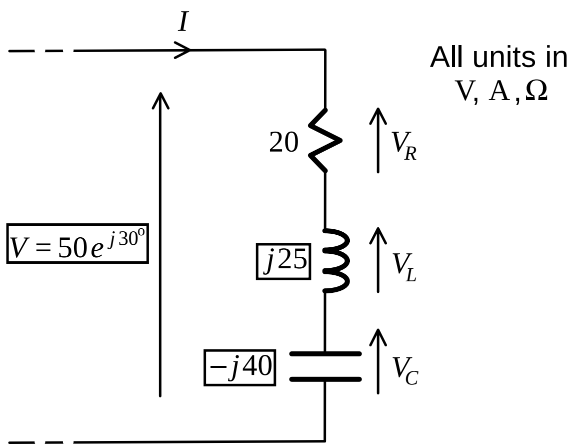
Q.4

Impedances and phasors

$v(t) = 50\sqrt{2}\cos(1250t + 30^0)\ \text{V} \;\Rightarrow\; V = 50e^{j30^0}\ \text{V}$.

Impedance of inductor $= j(1250)(0.02) = j25\Omega$

Impedance of capacitor $= \dfrac{1}{j(1250)(0.00002)} = -j40\Omega$



Total impedance $= 20 + j25 - j40 = 20 - j15 = \sqrt{20^2 + 15^2}\, e^{j\tan^{-1}\left(\frac{-15}{20}\right)} = 25e^{-36.9^0}\ \Omega$

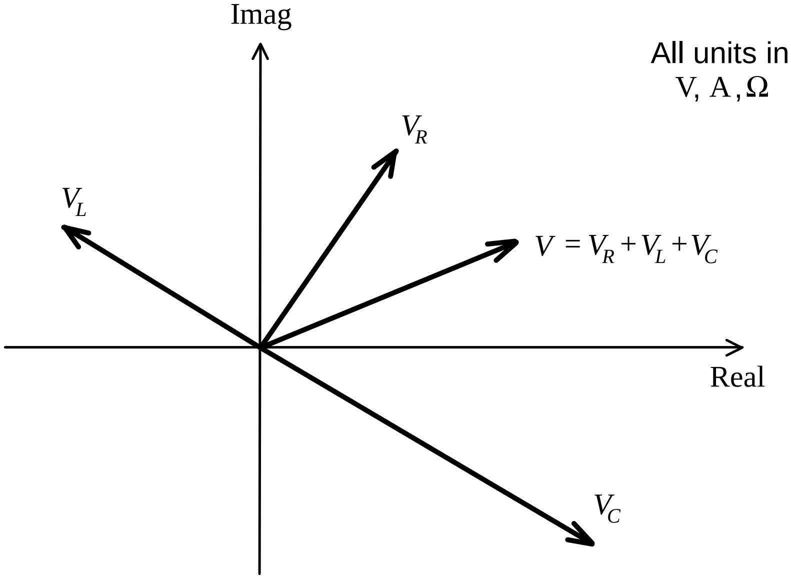
$$I = \frac{V}{20 + j25 - j40} = \frac{50e^{j30^0}}{25e^{j-36.9^0}} = 2e^{j66.9^0}\ \text{A}$$

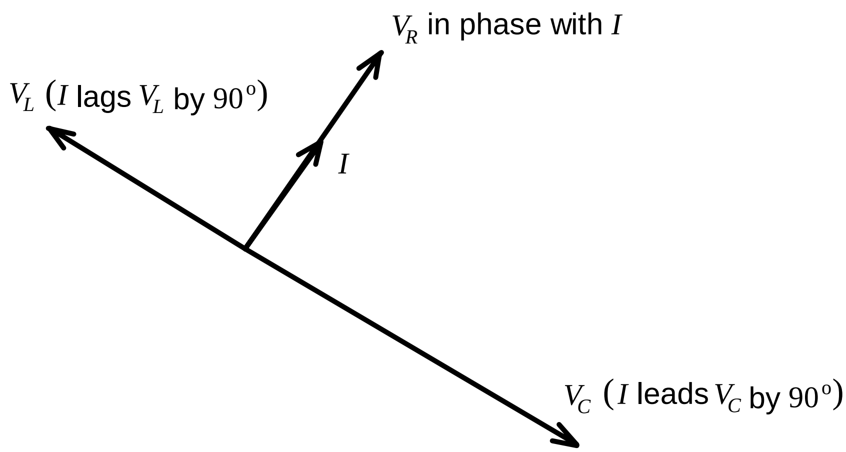
$$V_R = 20I = (20)\left(2e^{j66.9^0}\right) = 40e^{j66.9^0}\ \text{V}$$

$$V_L = j25I = \left(25e^{j90^0}\right)\left(2e^{j66.9^0}\right) = 50e^{j156.9^0}\ \text{V}$$

$$V_C = -j40I = \left(40e^{-j90^0}\right)\left(2e^{j66.9^0}\right) = 80e^{-j23.1^0}\ \text{V}$$

Phasor diagram





Q.5

Components

The impedances of series RL, RC and LC circuits are

$$Z_{RL} = R + j\omega L = R + j100\pi\, L$$

$$Z_{RC} = R + \frac{1}{j\omega C} = R - \frac{j}{100\pi C}$$

$$Z_{LC} = j\omega L + \frac{1}{j\omega C} = j\left(100\pi\, L - \frac{1}{100\pi C}\right)$$

$20 + j30$ must correspond to a series RL circuit with components:

$$R_1 + j100\pi\, L_1 = 20 + j30 \;\Rightarrow\; R_1 = 20\Omega \text{ and } L_1 = \frac{30}{100\pi} = 0.0955\,\text{H}$$

$10 - j15$ must correspond to a series RC circuit with components:

$$R_2 - \frac{j}{100\pi C_2} = 10 - j15 \;\Rightarrow\; R_2 = 10\Omega \text{ and } C_2 = \frac{1}{100\pi(15)} = 212.3\,\mu\text{F}$$

Circuit admittance

$$\text{Circuit impedance} = Z = (20 + j30)\,\|\,(10 - j15) = \frac{1}{\dfrac{1}{20+j30} + \dfrac{1}{10-j15}}$$

$$\text{Circuit admittance} = \frac{1}{Z} = \frac{1}{20+j30} + \frac{1}{10-j15} = \frac{20-j30}{20^2+30^2} + \frac{10+j15}{10^2+15^2}$$

$$= 0.0154 - j0.0231 + 0.0308 + j0.0462 = (0.0462 + j0.0231)\,\Omega^{-1}$$

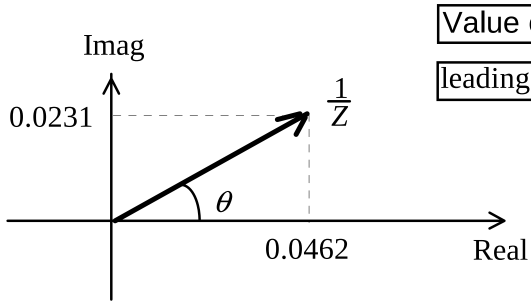
Power factor

$$\text{Power factor} = \left\langle \begin{array}{l} \text{value} = \cos[\text{Arg}(I) - \text{Arg}(V)] \\ \text{leading / lagging} = \begin{cases} \text{leading, Arg}(I) - \text{Arg}(V) > 0 \\ \text{lagging, Arg}(I) - \text{Arg}(V) < 0 \end{cases} \end{array} \right\rangle$$

$$\text{Arg}(I) - \text{Arg}(V) = \text{Arg}\left(\frac{I}{V}\right) = \text{Arg}\left(\frac{1}{Z}\right) = -\text{Arg}(Z)$$

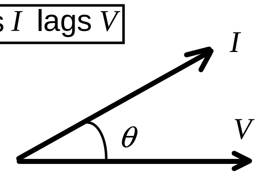
$$= \text{Arg}(0.0462 + j0.0231) = \tan^{-1}\left(\frac{0.0231}{0.0462}\right) = 0.464$$

$$\text{Power factor}\left\langle \begin{array}{c} \text{value} = \cos(0.464) = 0.894 \\ \text{leading / lagging} = \begin{cases} \text{leading, } 0.464 > 0 \\ \text{lagging, } 0.464 < 0 \end{cases} \end{array} \right\rangle = \left\langle \begin{array}{c} 0.894 \\ \text{leading} \end{array} \right\rangle$$



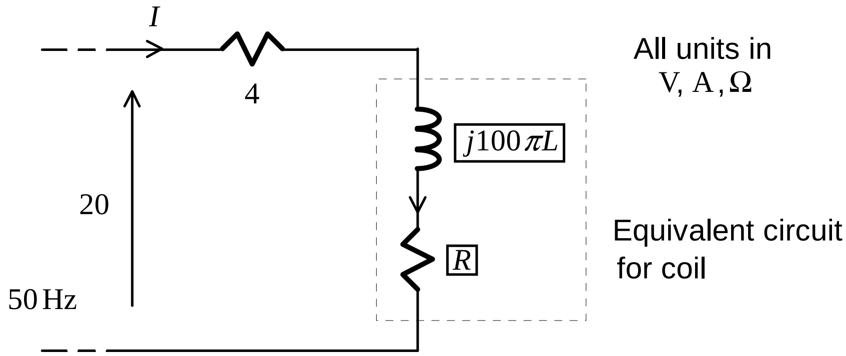
Value of p.f. $= \cos\theta$

leading p.f. as I lags V



F.6 Analiza Cir El c.a. II

Q.1 *Circuit diagram*



Main equations

$$\left|\text{Voltage across } 4\Omega\right| = |4I| = 9 \;\Rightarrow\; |I| = \frac{9}{4}$$

$$\left|\text{Voltage across coil}\right| = \left|I\left(R + j100\pi L\right)\right| = 14$$

$$\left|R + j314L\right| = \frac{14}{|I|} = 14\left(\frac{4}{9}\right) = 6.222 \;\Rightarrow\; R^2 + (314L)^2 = 38.72$$

$$\left|\text{Supply}\right| = \left|I\left(4 + R + j100\pi L\right)\right| = 20$$

$$\left|4 + R + j314L\right| = \frac{20}{|I|} = 20\left(\frac{4}{9}\right) = 8.889 \;\Rightarrow\; \left(R+4\right)^2 + (314L)^2 = 79.01$$

Component values

$$\left[\left(R+4\right)^2 + (314L)^2\right] - \left[R^2 + (314L)^2\right] = 79.01 - 38.72$$

$$8R + 16 = 40.29 \;\Rightarrow\; R = \frac{40.29 - 16}{8} = 3.04\,\Omega$$

$$R^2 + (314L)^2 = 38.72 \;\Rightarrow\; (314L)^2 = 38.72 - 3.04^2 = 29.48 \;\Rightarrow\; L = \frac{\sqrt{29.48}}{314} = 0.0173\,\text{H}$$

Power and power factor

$$\text{Power absorbed by coil} = |I|^2 R = \left(\frac{9}{4}\right)^2 (3.04) = 15.4\,\text{W}$$

$$\text{Power factor} = \left\langle \begin{array}{l} \text{value} = \cos\left[\text{Arg}(\text{current}) - \text{Arg}(\text{voltage})\right] \\ \text{leading / lagging} = \begin{cases} \text{leading, Arg}(\text{current}) - \text{Arg}(\text{voltage}) > 0 \\ \text{lagging, Arg}(\text{current}) - \text{Arg}(\text{voltage}) < 0 \end{cases} \end{array} \right\rangle$$

$$\text{Arg}(\text{current}) - \text{Arg}(\text{voltage}) = \text{Arg}\left(\frac{\text{current}}{\text{voltage}}\right)$$

$$= \text{Arg}\left(\frac{1}{\text{Impedance}}\right) = -\text{Arg}(\text{Impedance})$$

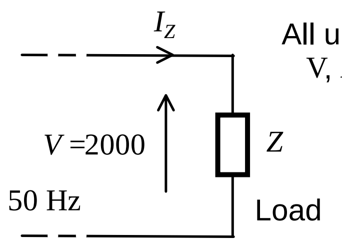
$$\text{Power factor} = \left\langle \begin{array}{l} \text{value} = \cos\left[\operatorname{Arg}(\text{impedance})\right] \\ \text{leading / lagging} = \begin{cases} \text{leading}, \operatorname{Arg}(\text{impedance}) < 0 \\ \text{lagging}, \operatorname{Arg}(\text{impedance}) > 0 \end{cases} \end{array} \right\rangle$$

$$\operatorname{Arg}(\text{impedance}) = \operatorname{Arg}(R + 4 + j314L) = \operatorname{Arg}(3.04 + 4 + j314 \times 0.0173)$$

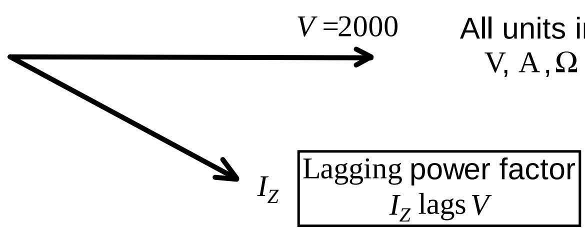
$$= \operatorname{Arg}(7.04 + j5.43) = \tan^{-1}\left(\frac{5.43}{7.04}\right) = 0.657$$

$$\text{Power factor} = \left\langle \begin{array}{l} \text{value} = \cos(0.657) = 0.792 \\ \text{leading / lagging} = \begin{cases} \text{leading}, 0.657 < 0 \\ \text{lagging}, 0.657 > 0 \end{cases} \end{array} \right\rangle = \left\langle \begin{array}{l} 0.792 \\ \text{lagging} \end{array} \right\rangle$$

Q.2 *Load current*



$$\text{Load power factor} = \frac{\text{actual power}}{\text{apparent power}} \Rightarrow 0.5 = \frac{10000}{2000|I_Z|} \Rightarrow |I_Z| = 10\,\text{A}$$



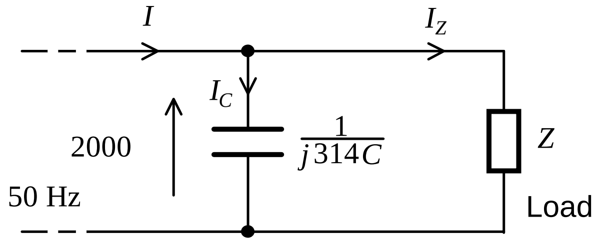
$$0.5\,\text{lagging load p.f.} \Rightarrow \left\langle \begin{array}{l} \cos\left[\operatorname{Arg}(I_Z) - \operatorname{Arg}(V)\right] = 0.5 \\ \operatorname{Arg}(I_Z) - \operatorname{Arg}(V) < 0 \end{array} \right\rangle$$

$$\Rightarrow \left\langle \begin{array}{l} \operatorname{Arg}(I_Z) = \pm\cos^{-1}(0.5) = \pm 1.05 \\ \operatorname{Arg}(I_Z) < 0 \end{array} \right\rangle$$

$$\Rightarrow \operatorname{Arg}(I_Z) = -1.05$$

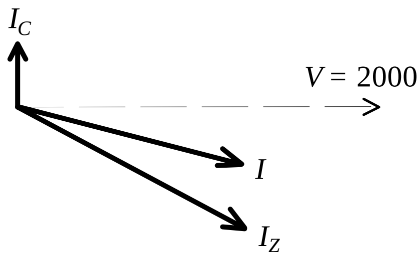
$$I_Z = |I_Z| e^{j\operatorname{Arg}[I_Z]} = 10 e^{-j1.05}\,\text{A}$$

Power factor improvement



$$I_C = (j314C)(2000) = j628000C$$

$$I = I_Z + I_C = 10e^{-j1.05} + j628000C = 5 + j(62800C - 8.66)$$



0.9 lagging overall p.f. $\Rightarrow \left\langle \begin{matrix} \cos[\text{Arg}(I) - \text{Arg}(V)] = 0.9 \\ \text{Arg}(I) - \text{Arg}(V) < 0 \end{matrix} \right\rangle \Rightarrow \left\langle \begin{matrix} \text{Arg}(I) = \pm 0.451 \\ \text{Arg}(I) < 0 \end{matrix} \right\rangle$

$$\Rightarrow \text{Arg}(I) = -0.451 \Rightarrow \tan^{-1}\left(\frac{628000C - 8.66}{5}\right) = -0.451$$

$$\Rightarrow C = \frac{8.66 - 5\tan(-0.451)}{628000} = 0.0000176\,\text{F} = 17.6\,\mu\ \text{F}$$

unity overall p.f. $\Rightarrow \left\langle \begin{matrix} \cos[\text{Arg}(I) - \text{Arg}(V)] = 1 \\ \text{Arg}(I) - \text{Arg}(V) = 0 \end{matrix} \right\rangle \Rightarrow \text{Arg}(I) = 0$

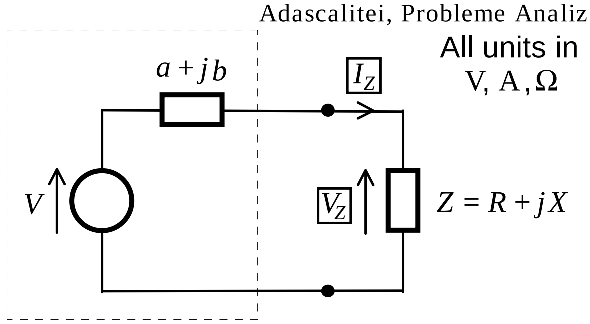
$$\Rightarrow \tan^{-1}\left(\frac{628000C - 8.66}{5}\right) = 0 \Rightarrow C = \frac{8.66}{628000} = 0.0000138\,\text{F} = 13.8\,\mu\ \text{F}$$

0.8 leading overall p.f. $\Rightarrow \left\langle \begin{matrix} \cos[\text{Arg}(I) - \text{Arg}(V)] = 0.8 \\ \text{Arg}(I) - \text{Arg}(V) > 0 \end{matrix} \right\rangle \Rightarrow \left\langle \begin{matrix} \text{Arg}(I) = \pm 0.644 \\ \text{Arg}(I) > 0 \end{matrix} \right\rangle$

$$\Rightarrow \text{Arg}(I) = 0.644 \Rightarrow \tan^{-1}\left(\frac{628000C - 8.66}{5}\right) = 0.644$$

$$\Rightarrow C = \frac{8.66 + 5\tan(0.644)}{628000} = 0.0000078\,\text{F} = 7.8\,\mu\ \text{F}$$

Q.3 *Power*



Electrical system

$$I_Z = \frac{V}{(a+jb)+(R+jX)} = \frac{V}{(a+R)+j(b+X)}$$

$$V_Z = I_Z(R+jX)$$

Power absorbed $= p = \mathrm{Re}\left[I_Z^* V_Z\right] = \mathrm{Re}\left[|I_Z|^2 (R+jX)\right]$

$$= |I_Z|^2 \,\mathrm{Re}[R+jX] = R|I_Z|^2$$

$$= R\left|\frac{V}{(a+R)+j(b+X)}\right|^2 = \frac{R|V|^2}{(a+R)^2+(b+X)^2}$$

Maximum power transfer

For maximum p, the denominator should be as small as possible. As the numerator does not depend on X and the smallest value for $(b+X)$ is 0, maximum power will be absorbed if

$$X = -b$$

so that

$$p = \frac{|V|^2 R}{(a+R)^2}$$

Differentiating:

$$\frac{dp}{dR} = |V|^2\left[\frac{1}{(a+R)^2} - \frac{2R}{(a+R)^3}\right] = |V|^2\left[\frac{a-R}{(a+R)^3}\right]$$

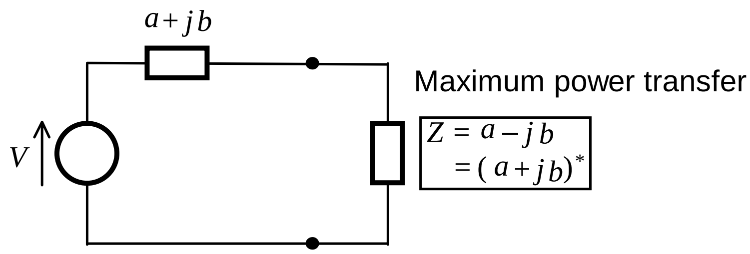
Thus, maximum p occurs when

$$R = a$$

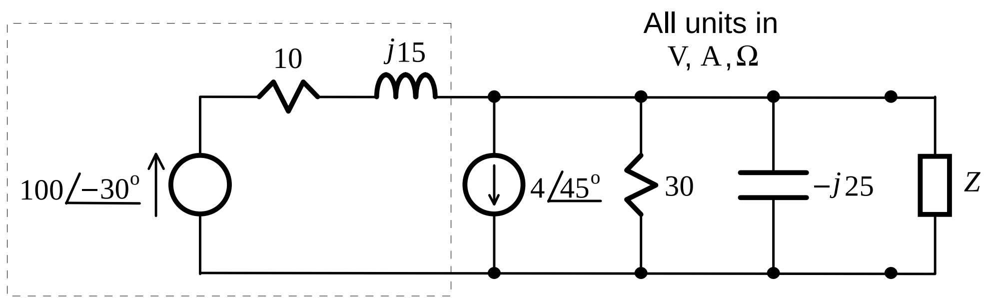
and the maximum power transferable is

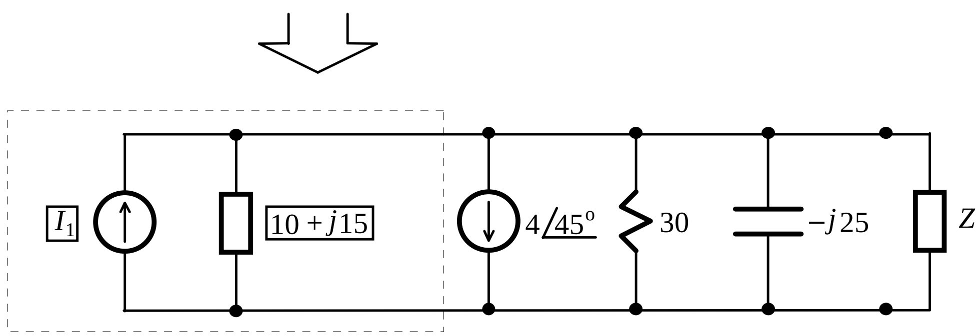
$$p = \frac{|V|^2 R}{(R+R)^2} = \frac{|V|^2}{4R} = \frac{|V|^2}{4a}\,\mathrm{W}$$

In general, maximum power transfer occurs when the load impedance is equal to the conjugate of the Thevenin's or Norton's impedance. When this occurs, the total impedance is purely resistive and the current and voltage in the circuit are in phase:

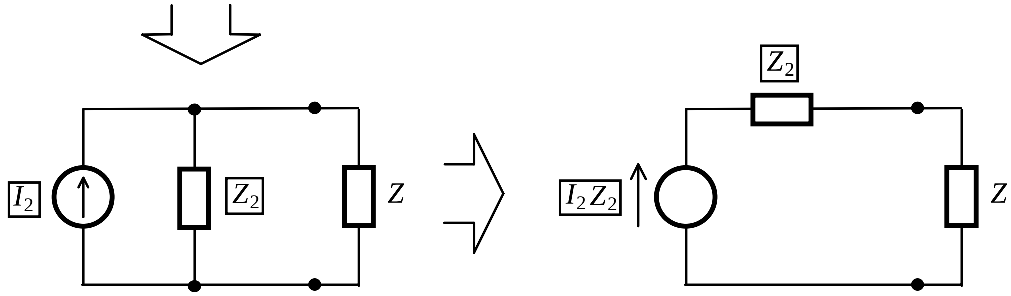


Q.4 *Norton's and Thevenin's equivalent circuit*





$$I_1 = \frac{100e^{-j30^0}}{10+j15} = \frac{100e^{-j30^0}}{18e^{j56.3^0}} = 5.55e^{-j86.3^0} = 0.358 - j5.54$$



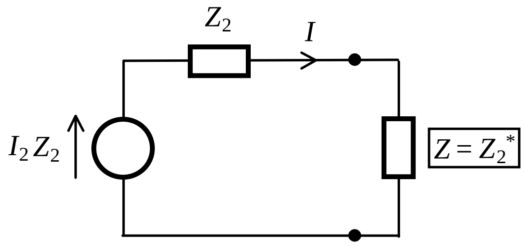
$$Z_2 = -j25 \,||\, 30 \,||\, (10+j15) = \frac{1}{\frac{1}{-j25}+\frac{1}{30}+\frac{1}{10+j15}} = \frac{1}{\frac{1}{-j25}+\frac{1}{30}+\frac{10-j15}{10^2+15^2}}$$

$$= \frac{1}{0.0641 - j0.00615} = \frac{1}{0.0644e^{-j5.48^0}} = 15.5e^{j5.48^0}$$

$$I_2 = 0.359 - j5.55 - 4e^{j45^0} = -2.47 - j8.36 = 8.72e^{-j106^0}$$

Maximum power transfer

From the previous problem, this occurs when



$$Z = Z_2^* = \left(15.5e^{j5.48^0}\right)^* = 15.5e^{-j5.48^0}$$

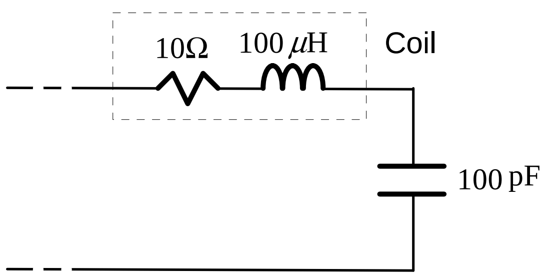
Total impedance $= Z + Z_2 = Z_2^* + Z_2 = 15.5e^{j5.48^0} + 15.5e^{-j5.48^0} = 2\left[15.5\cos\left(5.48^0\right)\right]$

$$I = \frac{I_2 Z_2}{\left(Z + Z_2\right)} = \frac{8.72e^{-j106^0}\,15.5e^{j5.48^0}}{2\left[15.5\cos\left(5.48^0\right)\right]} = \frac{8.72e^{-j101^0}}{2\cos\left(5.48^0\right)}$$

Thus, the maximum power transferable is

$$\mathrm{Re}\left[I^*(IZ)\right] = |I|^2\,\mathrm{Re}\left[Z_2^*\right]$$
$$= \left|\frac{8.72e^{-j101^0}}{2\cos\left(5.48^0\right)}\right|^2 \mathrm{Re}\left[15.5e^{-j5.48^0}\right] = \left|\frac{8.72}{2\cos\left(5.48^0\right)}\right|^2 15.5\cos\left(5.48^0\right)$$
$$= \frac{8.72^2 \times 15.5}{4\cos\left(5.48^0\right)} = 297\,\mathrm{W}$$

Q.5



Resonant frequency $= \dfrac{1}{2\pi\sqrt{\left(100\times10^{-6}\right)\left(100\times10^{-12}\right)}} = 1.59\,\mathrm{MHz}$

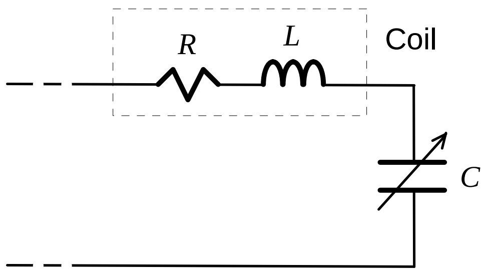
Q factor $= \dfrac{2\pi\left(1.59\times10^{6}\right)\left(100\times10^{-6}\right)}{10} = 100$

Since the Q factor is large, the circuit is bandpass in nature with

$3\mathrm{dB}$ cutoff frequencies $\approx 1.59\left(1 \pm \dfrac{1}{200}\right)\mathrm{MHz}$

$$\text{Bandwidth} \approx \frac{1.59\times10^6}{100} = 15.9\,\text{kHz}$$

Q.6



$$C = 20 \cdots 500\,\text{pF} \;\Rightarrow\; \text{resonant frequency} = \frac{1}{2\pi\sqrt{L\left(500\times10^{-12}\right)}} \cdots \frac{1}{2\pi\sqrt{L\left(20\times10^{-12}\right)}}$$

$$= \frac{7.12}{\sqrt{L}} \cdots \frac{35.6}{\sqrt{L}}\,\text{kHz}$$

For the lowest tunable frequency to be $666\,\text{kHz}$:

$$666 = \frac{7.12}{\sqrt{L}} \;\Rightarrow\; L = \left(\frac{7.12}{666}\right)^2 = 0.114\,\text{mH}$$

The highest tunable frequency is then

$$\frac{35.6}{\sqrt{L}} = \frac{35.6}{\sqrt{0.114\times10^{-3}}} = 3.42\,\text{MHz}$$