13.2. ECUATIILE SI PARAMETRII CUADRIPOLILOR LINIARI, PASIVI SI RECIPROCI

Dintre cele patru variabile $\underline{I}_1, \underline{I}_2, \underline{U}_1, \underline{U}_2$ care caracterizeaza interactiunea cuadripolului cu exteriorul, numai doua sunt independente din punctul de vedere al structurii interioare a cuadripolului.

Daca de exemplu, se aplica la borne tensiunile $\underline{U}_1, \underline{U}_2$ cunoscute, teoremele lui Kirchhoff permit determinarea unica a curentilor $\underline{I}_1, \underline{I}_2$. Exista deci, intre aceste patru variabile doua relatii de forma:

$$F_1(\underline{U}_1, \underline{U}_2, \underline{I}_1, \underline{I}_2) = 0; \qquad F_2(\underline{U}_1, \underline{U}_2, \underline{I}_1, \underline{I}_2) = 0 \qquad (13.4)$$

numite ecuatiile cuadripolului sub forma implicita, a caror cunoastere e suficienta pentru studiul comportarii cuadripolului in reteaua mai mare din care face parte.

Cuadripolul fiind prin ipoteza liniar si pasiv, aceste ecuatii sunt neaparat liniare si omogene.

Ecuatiile cuadripolilor (13.4) au diferite forme explicite, obtinute alegând câte o anumita pereche de variabile ca variabile independente; $\underline{U}_2$ si $\underline{I}_2$, $\underline{U}_1$ si $\underline{I}_1$, $\underline{I}_1$ si $\underline{I}_2$, $\underline{U}_1$ si $\underline{U}_2$ etc. In aceste forme explicite, celelalte doua variabile sunt exprimate ca functii liniare si omogene de cele doua variabile independente, cei patru coeficienti complecsi ai acestor functii numindu-se parametrii cuadripolului sau constantele cuadripolului.

Caracterul pasiv al cuadripolului (asociat valorilor pozitive sau nule ale rezistentelor laturilor lui) mai impune satisfacerea conditiei ca puterea activa totala primita de cuadripol sa nu fie negativa, oricare ar fi valorile variabilelor independente:

$$\underline{P} = \underline{P}_1 - \underline{P}_2 = \mathrm{Re}\left\{\underline{U}_1 \underline{I}_1^x - \underline{U}_2 \underline{I}_2^x\right\} \geq 0 \qquad (13.5)$$

egalitatea corespunzând cuadripolilor nedisipativi.

Cei trei parametri complecsi independenti nu pot avea deci valori arbitrare, ci numai valorile compatibile cu conditia de pasivitate (13.5).

13.2.1. Forma fundamentala a ecuatiilor cuadripolului si parametrii fundamentali

Deoarece functiunea cea mai importanta a cuadripolilor e aceea de element al unui lant de transmisiune a energiei electromagnetice sau a semnalelor electromagnetice, forma fundamentala a ecuatiilor cuadripolilor e aceea in care marimile de intrare $\underline{U}_1$, $\underline{I}_1$ sunt exprimate in functiune de marimile de iesire $\underline{U}_2, \underline{I}_2$, prin relatii liniare si omogene de forme:

$$\begin{aligned} \underline{U}_1 &= \underline{A}\underline{U}_2 + \underline{B}\underline{I}_2 \\ \underline{I}_1 &= \underline{C}\underline{U}_2 + \underline{D}\underline{I}_2 \end{aligned} \qquad (13.6)$$

Coeficientii $\underline{A}$, $\underline{B}$, $\underline{C}$, $\underline{D}$ se numesc parametrii fundamentali ai cuadripolului. $\underline{A}$ si $\underline{D}$ sunt adimensionali, $\underline{B}$ e o impedanta, iar $\underline{C}$ o admitanta. Parametrii fundamentali au urmatoarele interpretari experimentale.

$$\underline{A} = \left(\frac{\underline{U}_1}{\underline{U}_2}\right)_{\underline{I}_2 = 0} = \text{raportul de transformare a tensiunilor la mersul in gol;}$$

$$\underline{B} = \frac{1}{\left(\frac{\underline{I}_2}{\underline{U}_1}\right)_{\underline{U}_2 = 0}} = \text{valoarea inversa a admitantei de transfer de scurtcircuit;}$$

$$\underline{C} = \left(\frac{1}{\frac{\underline{U}_2}{\underline{I}_1}} \right)_{\underline{I}_2=0} = \text{valoarea inversa a impedantei de transfer la mers in gol;} \quad (13.7)$$

$$\underline{D} = \left(\frac{\underline{I}_1}{\underline{I}_2} \right)_{\underline{U}_2=0} = \text{raportul de transformare al curentilor la mersul in scurtcircuit.}$$

intre acesti parametri, conditia de reciprocitate devine:

$$\Delta = \underline{A}\underline{D} - \underline{B}\underline{C} = 1 \quad (13.8)$$

Se observa: Δ e chiar determinantul relatiei (13.6) si conform cu relatia (13.8) acest determinant nu poate fi nul. Ecuatiile (13.6) au deci solutie unica, daca sunt explicite in raport cu $\underline{U}_2$ si $\underline{I}_2$. Folosind regula lui Cramer, se obtine o alta forma fundamentala a ecuatiilor, in care marimile de iesire $\underline{U}_2, \underline{I}_2$ sunt exprimate in functie de cele de intrare $\underline{U}_1, \underline{I}_1$

$$\begin{aligned} \underline{U}_2 &= \underline{D}\underline{U}_1 - \underline{B}\underline{I}_1 \\ \underline{I}_2 &= -\underline{C}\underline{U}_1 + \underline{A}\underline{I}_1 \end{aligned} \quad (13.9)$$

13.2.2. Parametrii impedanta

Daca ecuatiile fundamentale (13.6) se pot explicita in raport cu tensiunile $\underline{U}_1, \underline{U}_2$ (adica daca C≠0) se obtine o alta forma a ecuatiilor cuadripolilor liniari si pasivi.

$$\begin{cases} \underline{U}_1 = \underline{Z}_{11}\underline{I}_1 + \underline{Z}_{12}\underline{I}_2 \\ \underline{U}_2 = \underline{Z}_{21}\underline{I}_1 + \underline{Z}_{22}\underline{I}_2 \end{cases} \quad (13.10)$$

in care, coeficientii sunt parametrii impedanta a valorii polului.

$$\underline{Z}_{11} = \left(\frac{\underline{U}_1}{\underline{I}_1} \right)_{\underline{I}_2=0} = \frac{\underline{A}}{\underline{C}} = \text{impedanta de intrare la mers in gol.}$$

$$\underline{Z}_{12} = \left(\frac{\underline{U}_1}{\underline{I}_2} \right)_{\underline{I}_1=0} = \frac{\underline{A}\underline{D} - \underline{B}\underline{C}}{\underline{C}} \quad (13.11)$$

$$\underline{Z}_{21} = \left(\frac{\underline{U}_2}{\underline{I}_1} \right)_{\underline{I}_2=0} = \frac{1}{\underline{C}} = \text{impedanta de transfer la mers in gol.}$$

$$\underline{Z}_{22} = \left(\frac{\underline{U}_2}{\underline{I}_2} \right)_{\underline{I}_1=0} = -\frac{\underline{D}}{\underline{C}}.$$

Tinând seama de aceste expresii, conditia de reciprocitate, exprimata cu parametrii impedanta, se scrie:

$$\underline{Z}_{12} = -\underline{Z}_{21} \quad (13.12)$$

$$\Delta_Z = \underline{Z}_{11}\underline{Z}_{22} - \underline{Z}_{12}\underline{Z}_{21} = \underline{Z}_{11}\underline{Z}_{22} + \underline{Z}_{12}{}^2$$

13.2.3. Parametrii admitanta

Daca ecuatiile fundamentale (13.6) se pot explicita in raport cu curentii $\underline{I}_1, \underline{I}_2$ (adica daca $\underline{B} \neq 0$) se obtine o alta forma a ecuatiilor cuadripolilor liniari si pasivi.

$$\begin{cases} \underline{I}_1 = \underline{Y}_{11}\underline{U}_1 + \underline{Y}_{12}\underline{U}_2 \\ \underline{I}_2 = \underline{Y}_{21}\underline{U}_1 + \underline{Y}_{22}\underline{U}_2 \end{cases} \tag{13.13}$$

in care coeficientii sunt parametrii admitanta ai cuadripolului.

$$\underline{Y}_{11} = \left(\frac{\underline{I}_1}{\underline{U}_2}\right)_{\underline{U}_2=0} = \frac{\underline{D}}{\underline{B}} = \text{admitanta de intrare de scurtcircuit.}$$

$$\underline{Y}_{12} = \left(\frac{\underline{I}_1}{\underline{U}_1}\right)_{\underline{U}_1=0} = -\frac{\underline{A}\underline{D} - \underline{B}\underline{C}}{\underline{B}}$$

$$\underline{Y}_{21} = \left(\frac{\underline{I}_2}{\underline{U}_1}\right)_{\underline{U}_2=0} = \frac{1}{\underline{B}} = \text{admitanta de transfer de scurtcircuit.}$$

$$\underline{Y}_{22} = \left(\frac{\underline{I}_2}{\underline{U}_2}\right)_{\underline{U}_1=0} = -\frac{\underline{A}}{\underline{B}} \tag{13.14}$$

Conditia de reciprocitate se scrie: $\underline{Y}_{12} = -\underline{Y}_{21}$ (13.15)

iar

$$\Delta_y = \underline{Y}_{11}\underline{Y}_{22} - \underline{Y}_{12}\underline{Y}_{21} = \underline{Y}_{11}\underline{Y}_{22} + \underline{Y}_{12}^2 \tag{13.16}$$

13.2.4. Relatii intre diferite categorii de parametrii

Daca $\Delta_Z \neq 0$, parametrii admitanta se pot exprima in functie de parametrii impedanta prin relatiile:

$$\underline{Y}_{11} = \frac{\underline{Z}_{22}}{\underline{\Delta}_Z};\quad \underline{Y}_{12} = \frac{\underline{Z}_{12}}{\underline{\Delta}_Z};\quad \underline{Y}_{21} = \frac{\underline{Z}_{21}}{\underline{\Delta}_Z};\quad \underline{Y}_{22} = \frac{\underline{Z}_{11}}{\Delta_Z} \tag{13.17}$$

iar daca $\Delta_y \neq 0$, parametrii impedanta se pot exprima in functie de parametrii admitanta prin relatiile:

$$\underline{Z}_{11} = \frac{\underline{Y}_{22}}{\underline{\Delta}_y};\quad \underline{Z}_{12} = \frac{\underline{Y}_{12}}{\underline{\Delta}_y};\quad \underline{Z}_{21} = -\frac{\underline{Y}_{21}}{\underline{\Delta}_y};\quad \underline{Z}_{22} = \frac{\underline{Y}_{11}}{\Delta_y} \tag{13.18}$$

Se observa imediat ca $\Delta_z \cdot \Delta_y = 1$ (13.19)

Daca se dau parametrii admitanta, respectiv impedanta si daca $\underline{Y}_{21} \neq 0$, respectiv $\underline{Z}_{21} \neq 0$, se pot calcula parametrii fundamentali:

$$\underline{A} = -\frac{\underline{Y}_{22}}{\underline{Y}_{21}} = \frac{\underline{Z}_{11}}{\underline{Z}_{21}};\quad \underline{B} = \frac{1}{\underline{Y}_{21}} = -\frac{\Delta_Z}{\underline{Z}_{21}};\quad \underline{C} = -\frac{\Delta_y}{\underline{Y}_{21}} = \frac{1}{\underline{Z}_{21}} \tag{13.20}$$

$$\underline{D} = \frac{\underline{Y}_{11}}{\underline{Y}_{21}} = -\frac{\underline{Z}_{22}}{\underline{Z}_{21}}$$

Se numeste cuadripol degenerat un cuadripol pentru care unul din determinantii Δ_z respectiv Δ_y este nul.

13.2.5. Cuadripoli simetrici

Un cuadripol poate fi alimentat pe la bornele secundare, care constituie in acest caz, bornele de intrare si poate debita pe la bornele primare, care constituie, la acest caz bornele de iesire. Aceasta este alimentarea inversa a cuadripolului fig. 13.4 care corespunde in ecuatii, schimbarii sensului de referinta a curentului.

$$\underline{U}'_1 = \underline{U}_1; \quad \underline{U}'_2 = \underline{U}_2; \quad \underline{I}'_1 = -\underline{I}_1; \quad \underline{I}'_2 = -\underline{I}_2 \qquad (13.21)$$

Ecuatiile cuadripolului la alimentare inversa se obtin cu (13.21) introdusa in relatia (13.9).

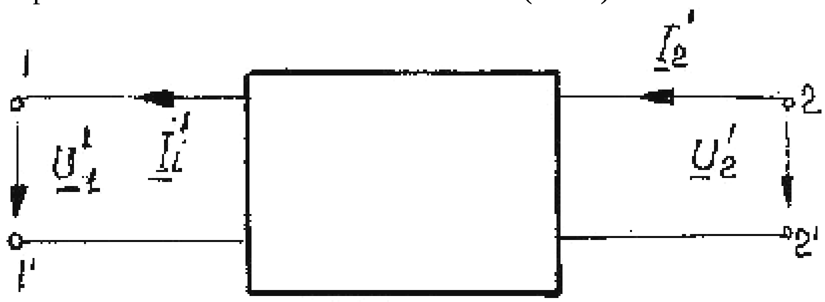

Fig. 13.4

$$\begin{aligned} \underline{U}'_2 &= \underline{D}\underline{U}'_1 = \underline{B}\underline{I}'_1 \\ \underline{I}'_2 &= \underline{C}\underline{U}'_1 + \underline{A}\underline{I}'_1 \end{aligned} \qquad (13.22)$$

Se numeste cuadripol simetric un cuadripol la care intervertirea portilor de intrare si de iesire nu afecteaza exteriorul. Pentru aceasta este necesar si suficient ca prin substitutia $1 \rightleftarrows 2$, ecuatiile la alimentarea inversa sa coincida cu ecuatia (13.6). Conditia de simetrie necesara si suficienta, rezulta a fi:

$$\underline{A} = \underline{D} \qquad (13.23)$$

Cu ajutorul parametrilor impedanta, respectiv admitanta, conditia de simetrie se scrie $\underline{Z}_{22} = -\underline{Z}_{11}$, respectiv $\underline{Y}_{22} = -\underline{Y}_{11}$ (13.24)

Un cuadripol simetric reciproc are numai parametri impedanta, iar conditia de reciprocitate se scrie

$$\underline{A}^2 = 1 + \underline{B}\underline{C} \qquad (13.25)$$