IV. CUADRIPOLI SI FILTRE ELECTRICE

CAP. 13.
CUADRIPOLI ELECTRICI

Breviar

a) Forma fundamentala a ecuatiilor cuadripolilor si parametrii fundamentali: Prima forma fundamentala:

$$\underline{U}_1 = \underline{A} \cdot \underline{U}_2 + \underline{B} \cdot \underline{I}_2$$
$$\underline{I}_1 = \underline{C} \cdot \underline{U}_2 + \underline{D} \cdot \underline{I}_2$$

A doua forma fundamentala:

$$\underline{U}_2 = \mathrm{D} \cdot \underline{U}_1 - \underline{B} \cdot \underline{I}_1$$
$$\underline{I}_2 = -\underline{C} \cdot \underline{U}_1 + \underline{A} \cdot \underline{I}_1$$

b) Parametrii fundamentali au urmatoarele interpretari experimentale:

$\underline{A} = \left(\dfrac{\underline{U}_1}{\underline{U}_2}\right)_{\underline{I}_2=0}$ – raportul de transformare al tensiunilor la functionarea in gol

$\underline{B} = \dfrac{1}{\left(\dfrac{\underline{I}_2}{\underline{U}_1}\right)_{\underline{U}_2=0}}$ – valoarea reciproca a admitantei de transfer la functionarea in scurtcircuit;

$\underline{C} = \dfrac{1}{\left(\dfrac{\underline{U}_2}{\underline{I}_1}\right)_{\underline{I}_2=0}}$ – valoarea reciproca a impedantei de transfer la functionarea in gol;

$\underline{D} = \left(\dfrac{\underline{I}_1}{\underline{I}_2}\right)_{\underline{U}_2=0}$ – raportul de transformare al curentilor la functionarea in scurtcircuit.

c) Marimi caracteristice ale cuadripolilor:

– impedanta primara de mers in gol:

$$\underline{Z}_{10} = \left(\frac{\underline{U}_1}{\underline{I}_1}\right)_{\underline{I}_2=0} = \frac{\underline{A} \cdot \underline{U}_{20}}{\underline{C} \cdot \underline{U}_{20}} = \frac{\underline{A}}{\underline{C}}$$

– impedanta primara de scurtcircuit:

$$\underline{Z}_{isc} = \left(\frac{\underline{U}_1}{\underline{I}_1}\right)_{\underline{U}_2=0} = \frac{\underline{B}}{\underline{D}}$$

– impedanta secundara de mers in gol:

$$\underline{Z}_{20} = \left(\frac{\underline{U}'_2}{\underline{I}'_2}\right)_{\underline{I}'_1=0} = \frac{\underline{D} \cdot \underline{U}'_{10}}{\underline{C} \cdot \underline{U}'_{10}} = \frac{\underline{D}}{\underline{C}}$$

– impedanta secundara de scurtcircuit:

$$\underline{Z}_{2sc} = \left(\frac{\underline{U}'_2}{\underline{I}'_2}\right)_{\underline{U}'_1=0} = \frac{\underline{B}\cdot\underline{I}'_1}{\underline{A}\cdot\underline{I}'_1} = \frac{\underline{B}}{\underline{A}}$$

d) Parametrii impedanta si cuadripolului:

Daca parametrul fundamental $\underline{C}$ este diferit de zero ecuatiile fundamentale ale cuadripolului se pot explicita in raport cu tensiunile $\underline{U}_1$, $\underline{U}_2$ si se obtine urmatoarea forma a ecuatiilor cuadripolilor liniari si pasivi:

$\underline{U}_1 = \underline{Z}_{11}\cdot\underline{I}_1 + \underline{Z}_{12}\cdot\underline{I}_2$ si

$\underline{U}_2 = \underline{Z}_{21}\cdot\underline{I}_1 + \underline{Z}_{22}\cdot\underline{I}_2$, unde:

$$\underline{Z}_{11} = \left(\frac{\underline{U}_1}{\underline{I}_1}\right)_{\underline{I}_2=0} = \frac{\underline{A}}{\underline{C}}$$

$$\underline{Z}_{12} = \left(\frac{\underline{U}_1}{\underline{I}_2}\right)_{\underline{I}_1=0} = \frac{\underline{A}\cdot\underline{D}-\underline{B}\cdot\underline{C}}{\underline{C}}$$

$$\underline{Z}_{21} = \left(\frac{\underline{U}_2}{\underline{I}_1}\right)_{\underline{I}_2=0} = \frac{1}{\underline{C}};$$

$$\underline{Z}_{22} = \left(\frac{\underline{U}_2}{\underline{I}_2}\right)_{\underline{I}_1=0} = -\frac{\underline{D}}{\underline{C}}$$

e) Parametrii admitanta ai cuadripolului.

Daca parametrul fundamental $\underline{B}$ este diferit de zero, in ecuatiile fundamentale se pot explicita curentii $\underline{I}_1, \underline{I}_2$, obtinându-se sistemul de ecuatii:

$\underline{I}_1 = \underline{Y}_{11}\underline{U}_1 + \underline{Y}_{12}\underline{U}_2$

$\underline{I}_2 = \underline{Y}_{21}\underline{U}_1 + \underline{Y}_{22}\underline{U}_2$

$$\underline{Y}_{11} = \left(\frac{\underline{I}_1}{\underline{U}_1}\right)_{\underline{U}_2=0} = \frac{\underline{D}}{\underline{B}}$$

$$\underline{Y}_{12} = \left(\frac{\underline{I}_1}{\underline{U}_2}\right)_{\underline{U}_1=0} = \frac{\underline{A}\cdot\underline{D}-\underline{B}\cdot\underline{C}}{\underline{B}}$$

$$\underline{Y}_{21} = \left(\frac{\underline{I}_2}{\underline{U}_1}\right)_{\underline{U}_2=0} = \frac{1}{\underline{B}}$$

$$\underline{Y}_{22} = \left(\frac{\underline{I}_2}{\underline{U}_2}\right)_{\underline{U}_1=0} = -\frac{\underline{A}}{\underline{B}}$$

13.1. (R) Sa se determine constantele $\underline{A}, \underline{B}, \underline{C}, \underline{D}$ si impedanta caracteristica primara pentru cuadripolul din fig.13.1.a. Date numerice: R=10Ω; R_0=5Ω; X=10Ω; X_0=5Ω.

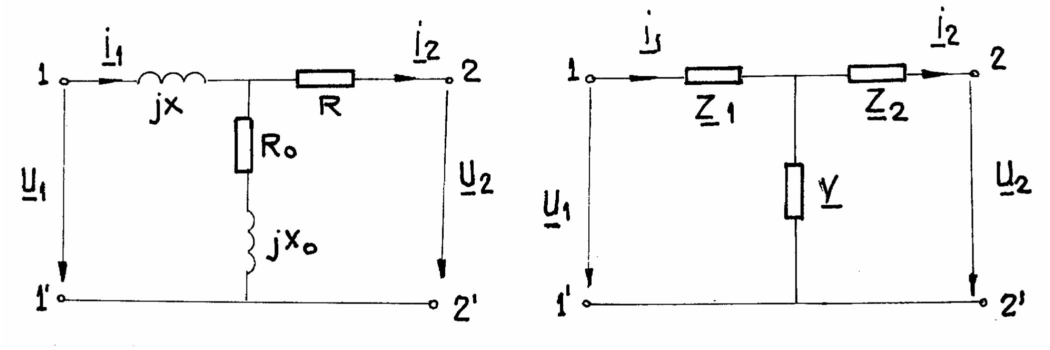

Fig. 13.1

Rezolvare: Cuadripolul dat fiind un cuadripol in T, se va considera schema generala (fig.13.1.b) a unui cuadripol T si se vor determina pentru aceasta schema constantele A, B, C, D.

Aplicând metoda curentilor ciclici se obtin ecuatiile:

$$\left(\underline{Z}_1 + \frac{1}{\underline{Y}}\right) \cdot \underline{I}_1 - \frac{1}{\underline{Y}} \cdot \underline{I}_2 = \underline{U}_1$$

$$-\frac{1}{\underline{Y}} \cdot \underline{I}_1 + \left(\underline{Z}_2 + \frac{1}{\underline{Y}}\right) \underline{I}_2 = -\underline{U}_2$$

deoarece $\underline{I}'_1 = \underline{I}_1; \underline{I}'_2 = \underline{I}_2$

Din ultima ecuatie: $\underline{I}_1 = \underline{Y} \cdot \underline{U}_2 + (1 + \underline{Z}_2 \cdot \underline{Y}) \cdot \underline{I}_2$

si, introducând in prima, se obtine:

$$\underline{U}_1 = (1 + \underline{Z}_1 \cdot \underline{Y}) \cdot \underline{U}_2 + (\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_1 \cdot \underline{Z}_2 \cdot \underline{Y}) \cdot \underline{I}_2$$

Prin urmare:

$$\underline{A} = 1 + \underline{Z}_1 \cdot \underline{Y}$$

$$\underline{B} = \underline{Z}_1 + \underline{Z}_2 + \underline{Z}_1 \underline{Z}_2 \cdot \underline{Y}$$

$$\underline{C} = \underline{Y}$$

$$\underline{D} = 1 + \underline{Z}_2 \cdot \underline{Y}$$

numeric, se gaseste:

$$\underline{Z}_1 = j \cdot X = j \cdot 10\Omega$$

$$\underline{Z}_2 = R = 10\Omega$$

$$\underline{Y} = \frac{1}{R_0 + j \cdot X_0} = \frac{1}{5 + 5j}$$

Deci:

$$\underline{A} = 1 + \underline{Z}_1 \cdot \underline{Y} = 1 + \frac{j \cdot 10}{5 + j \cdot 5} = 2 + j$$

$$\underline{B} = \underline{Z}_1 + \underline{Z}_2 + \underline{Z}_1 \cdot \underline{Z}_2 \cdot \underline{Y} = j \cdot 10 + 10 + j \cdot 10 \cdot 10 \cdot \frac{1}{5 + j \cdot 5} = (20 + j \cdot 20)\Omega$$

$$\underline{C} = \underline{Y} = \frac{1}{5 + j \cdot 5} = (0{,}1 - j \cdot 0{,}1)S$$

$$\underline{D} = 1 + \underline{Z}_2 \cdot \underline{Y} = 1 + 10 \cdot \frac{1}{5 + j \cdot 5} = 1 + 1 - j = 2 - j$$

Pentru calculul impedantei caracteristice primare se foloseste relatia:

$$\underline{Z}_{1c} = \frac{\underline{A} \cdot \underline{Z}_{1c} + \underline{B}}{\underline{C} \cdot \underline{Z}_{1c} + \underline{D}}$$

din care rezulta ecuatia:

$$\underline{C} \cdot \underline{Z}_{1c}^2 + (\underline{D} - \underline{A}) \cdot \underline{Z}_{1c} - \underline{B} = 0$$

pe care rezolvând-o se obtine:

$$\underline{Z}_{1c} = \frac{\underline{A} - \underline{D}}{2\underline{C}} \pm \sqrt{\frac{(\underline{A} - \underline{D})^2}{4\underline{C}^2} + \frac{\underline{B}}{\underline{C}}}$$

si numeric:

$$\underline{Z}_{1c} = \frac{2 + j - 2 + j}{2(0,1 - j \cdot 0,1)} \pm \sqrt{\frac{(2j)^2}{4(0,1 - j \cdot 0,1)^2} + \frac{20 + j \cdot 20}{0,1 - j \cdot 0,1}} =$$

$$= \begin{cases} 5(\sqrt{3} - 1) + j \cdot 5(1 + \sqrt{3}) \ \ \Omega \\ -5(\sqrt{3} + 1) - j \cdot 5(\sqrt{3} - 1) \ \ \Omega \end{cases}$$

Se retine numai prima valoare deoarece a doua are partea reala negativa si deci nu poate fi realizabila practic, deci:

$$\underline{Z}_{1c} = 5(\sqrt{3} - 1) + j \cdot 5(\sqrt{3} + 1) = (3,65 + j \cdot 13,65)\Omega$$

adica este formata dintr-o rezistenta de 3,65 Ω in serie cu o bobina ideala de inductivitate:

$L = \dfrac{13,65}{2 \cdot \pi \cdot 50} = \dfrac{13,65}{314} = 4,34 \cdot 10^{-2} H = 43,4$ mH in cazul frecventei de alimentare f=50 Hz.

13.2 (R) Sa se determine valorile elementelor schemelor echivalente in T, π si Γ ale unui cuadripol oarecare ai carui parametri $\underline{A}, \underline{C}, \underline{D}$ au valorile: $\underline{A}$=0,3+j·0,6; $\underline{C}$=(0,1+j·0,2); $\underline{D}$=0,5+j·0,5.

Rezolvare: Aplicând teorema a II-a lui Kirchhoff celor doua ochiuri independente ale schemei echivalente in T din fig.13.2.a, se obtin ecuatiile:

$$\underline{U}_1 = \underline{Z}_1 \cdot \underline{I}_1 + \frac{1}{Y}(\underline{I}_1 - \underline{I}_2)$$

$$-\underline{U}_2 = \underline{Z}_2 \cdot \underline{I}_2 - \frac{1}{\underline{Y}}(\underline{I}_1 - \underline{I}_2)$$

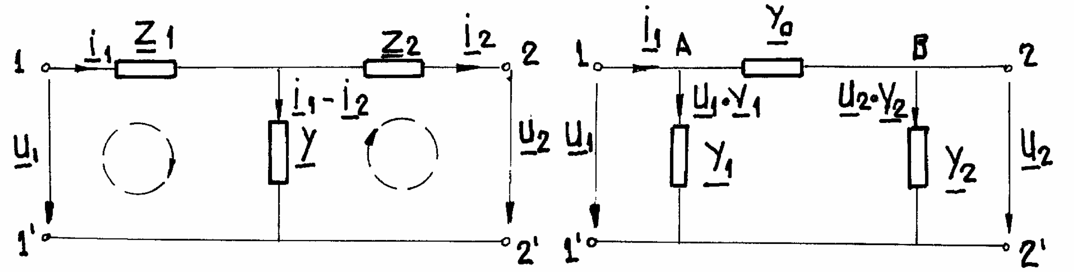

Fig. 13.2

Rezolvând acest sistem de ecuatii in raport cu marimile de la intrare ($\underline{I}_1$, U_1) se obtin ecuatiile:

$$\underline{I}_1 = \underline{U}_2 \cdot \underline{Y} + (\underline{Z}_2 \cdot \underline{Y} + 1) \cdot \underline{I}_2$$

$$\underline{U}_1 = (\underline{Z}_1 \cdot \underline{Y} + 1)\underline{U}_2 + (\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_1 \cdot \underline{Z}_2 \cdot \underline{Y})\underline{I}_2$$

de unde, prin comparatie cu ecuatiile:

$$\underline{U}_1 = \underline{A} \cdot \underline{U}_2 + \underline{B} \cdot \underline{I}_2$$

$$\underline{I}_1 = \underline{C} \cdot \underline{U}_2 + \underline{D} \cdot \underline{I}_2$$

se obtin coeficientii fundamentali:

$$\underline{A} = 1 + \underline{Z}_1 \cdot \underline{Y};$$

$$\underline{B} = \underline{Z}_1 \cdot \underline{Z}_2 \cdot \underline{Y} + \underline{Z}_1 + \underline{Z}_2;$$

$$\underline{C} = \underline{Y};$$

$$\underline{D} = \underline{1} + \underline{Z}_2 \cdot \underline{Y}$$

Rezulta de aici:

$$\underline{Y} = \underline{C};\ \underline{Z}_1 = (\underline{A} - 1)/\underline{C};\ Z_2 = (\underline{D} - 1)/\underline{C}$$

Numeric:

$$\underline{Z} = 1/\underline{Y} = (2 - j4)\Omega;\ \underline{Z}_1 = (1 + j4)\Omega;\ \underline{Z}_2 = (1 + j3)\Omega$$

In cazul cuadripolului in π, se aplica prima teorema a lui Kirchhoff la nodul B si a doua teorema a lui Kirchhoff pe ochiul care se inchide in lungul celor doua tensiuni la borne (fig.13.2.b) si se obtin ecuatiile:

$$\underline{I}_1 - \underline{U}_1 \cdot \underline{Y}_1 = \underline{I}_2 + \underline{U}_2 \cdot \underline{Y}_2$$

$$\underline{U}_1 = \frac{1}{\underline{Y}_0}(\underline{I}_2 + \underline{U}_2 \cdot \underline{Y}_2) + \underline{U}_2$$

sau:

$$\underline{U}_1 = (\underline{Y}_2 / \underline{Y}_0 + 1) \cdot \underline{U}_2 + 1/\underline{Y}_0 \cdot \underline{I}_2$$

$$\underline{I}_1 = \left(\underline{Y}_1 + \underline{Y}_2 + \frac{\underline{Y}_1 \underline{Y}_2}{\underline{Y}_0}\right) \cdot \underline{U}_2 + \left(\frac{\underline{Y}_1}{\underline{Y}_0} + 1\right)\underline{I}_2$$

de unde rezulta:

$$\underline{A} = 1 + \frac{\underline{Y}_2}{\underline{Y}_0};\ \underline{B} = 1/\underline{Y}_0$$

$$\underline{C} = \underline{Y}_1 + \underline{Y}_2 + \frac{\underline{Y}_1 \cdot \underline{Y}_2}{\underline{Y}_0};\ \underline{D} = \underline{1} + \underline{Y}_1 / \underline{Y}_0$$

sau:

$$\underline{Y}_0 = 1/\mathrm{B};\ \underline{Y}_1 = (\underline{D} - 1)/\underline{B};\ \underline{Y}_2 = (\underline{A} - 1)/\underline{B}$$

Numeric:

$$\underline{B} = \frac{\underline{A} \cdot \underline{D} - 1}{\underline{C}} = (-0{,}5 + j \cdot 5{,}5)\ \Omega$$

$$\underline{Y}_0 = (-0{,}0164 + j \cdot 0{,}180)\ \mathrm{S}$$

$$\underline{Y}_1 = (0,0985 + j \cdot 0,082)\ S$$
$$\underline{Y}_2 = (0,12 + j \cdot 0,116)\ S$$

Observatii:

a) Cuadripolul echivalent in T poate fi realizat (impedantele respective au partile reale pozitive) sub forma din fig.13.2.1;

b) Cuadripolul echivalent in π nu poate fi realizat (deoarece Y_0 are partea reala negativa).

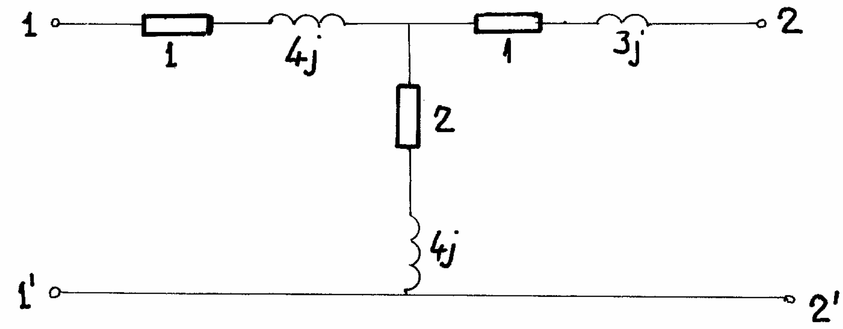

Fig. 13.2.1

Pentru determinarea elementelor cuadripolului echivalent in Γ (fig.13.2.2) se pot scrie ecuatiile:

$$\underline{I}_1 = \underline{I}_2 + \underline{Y} \cdot \underline{U}_2$$
$$\underline{U}_1 = \underline{U}_2 + \underline{Z} \cdot \underline{I}_1$$

sau:

$$\underline{U}_1 = \underline{U}_2 + \underline{Z}(\underline{I}_2 + \underline{Y} \cdot \underline{U}_2) = (1 + \underline{Z} \cdot \underline{Y}) \cdot \underline{U}_2 + \underline{Z} \cdot \underline{I}_2$$
$$\underline{I}_1 = \underline{Y} \cdot \underline{U}_2 + \underline{I}_2$$

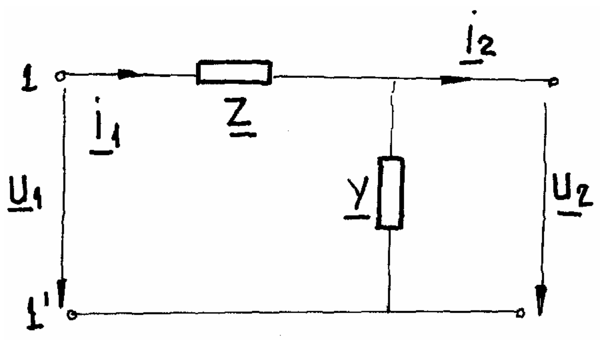

Fig. 13.2.2

Identificând cu ecuatiile:

$\underline{U}_1 = \underline{A} \cdot \underline{U}_2 + \underline{B} \cdot \underline{I}_2$ se obtine: $\underline{A} = 1 + \underline{Z} \cdot \underline{Y}; \underline{B} = \underline{Z}$

$\underline{I}_1 = \underline{C} \cdot \underline{U}_2 + \underline{D} \cdot \underline{I}_2$ $\qquad \underline{C} = \underline{Y}; \underline{D} = 1$

Deci: $\underline{Z} = \underline{B} = (0,5 + j \cdot 5,5)\ \Omega$

$$\underline{Y} = \underline{C} = (0,1 + j \cdot 0,2)\ S$$
$$\underline{Z}_0 = 1/Y = 1/(0,1 + j \cdot 0,2) = 10/(1 + 2j) = (2 - 4j)\Omega$$

realizarea acestui cuadripol este aratata in fig.13.2.3:

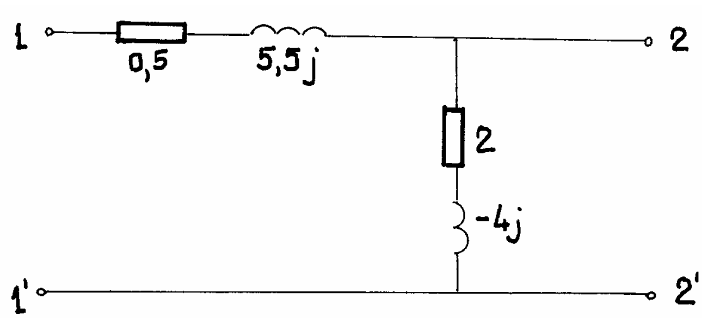

Fig. 13.2.3

13.3. (R) Pentru cuadripolul din fig.13.3, sa se determine coeficientii $\underline{Z}_{11}, \underline{Z}_{12}, \underline{Z}_{22}$, coeficientii $\underline{A}, \underline{B}, \underline{C}, \underline{D}$ precum si elementele cuadripolului in T echivalent. Date numerice: $X_1=\omega\cdot L_1=3\Omega$, $X_2=\omega\cdot L_2=2\Omega$; $R=1\Omega$, $X_M=\omega M=1\Omega$.

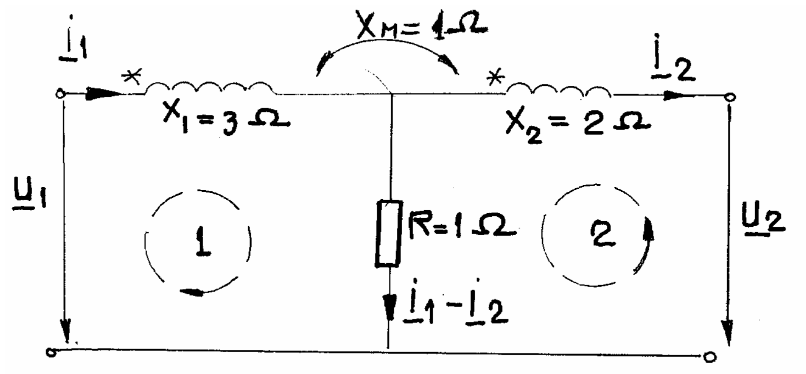

Fig.13.3

Rezolvare: Aplicând teorema a doua a lui Kirchhoff pe ochiurile 1 si 2, cu sensurile de parcurgere pozitiva a ochiurilor alese ca in figura, se obtine sistemul de ecuatii:

$$\begin{cases} \underline{U}_1 = j\cdot X_1\cdot \underline{I}_1 + R(\underline{I}_1 - \underline{I}_2) + j\cdot X_M\cdot \underline{I}_2 \\ \underline{U}_2 = -j\cdot X_2\cdot \underline{I}_2 + R(\underline{I}_1 - \underline{I}_2) - j\cdot X_M\cdot \underline{I}_1 \end{cases}$$

sau: $$\begin{cases} \underline{U}_1 = (R + j\cdot X_1)\cdot \underline{I}_1 + (-R + j\cdot X_M)\cdot \underline{I}_2 \\ \underline{U}_2 = (R - j\cdot X_M)\cdot \underline{I}_1 + (-R - j\cdot X_2)\cdot \underline{I}_2 \end{cases}$$

Rezulta: $\underline{Z}_{11} = R + j\cdot X_1$; $\underline{Z}_{12} = -\underline{Z}_{21} = -R + j\cdot X_M$;
$\underline{Z}_{22} = -R - j\cdot X_2$

sau numeric: $\underline{Z}_{11} = (1 + j \cdot 3)\Omega$; $\underline{Z}_{12} = (-1 + j)\Omega$; $\underline{Z}_{22} = (-1 - j2)\Omega$.

Rezolvând cea de-a doua ecuatie a sistemului de mai sus in raport cu $\underline{I}_1$ se obtine:

$$\underline{I}_1 = \frac{1}{1-j}\underline{U}_2 + \frac{1+2j}{1-j}\underline{I}_2$$

adica:

$$\underline{C} = 1/(1-j) = (0,5 + j \cdot 0,5)\ \text{S}$$
$$\underline{D} = (1+2j)/(1-j) = -0,5 + j \cdot 1,5$$

Pentru determinarea coeficientilor $\underline{A}$ si $\underline{B}$ se poate exprima $\underline{U}_1$ in functie de $\underline{U}_2$ si de $\underline{I}_2$ sau se folosesc formulele:

$$\underline{A} = \left(\frac{\underline{U}_1}{\underline{U}_2}\right)_{\underline{I}_2=0} = \frac{\underline{Z}_{11}}{\underline{Z}_{21}};\ \underline{C} = 1/\underline{Z}_{21};$$
$$\underline{B} = (\underline{A} \cdot \underline{D} - 1)/\underline{C} = \underline{Z}_{12} - (\underline{Z}_{11} \cdot \underline{Z}_{22})/\underline{Z}_{21}$$
$$\underline{D} = -\underline{Z}_{22}/\underline{Z}_{21}$$

Inlocuind numeric, se obtine:

$$\underline{A} = \frac{1+j3}{1-j} = -1 + j2;\ \underline{B} = \frac{(-1+2j)(-0,5 + j \cdot 1,5)}{0,5 + j \cdot 0,5} = (-6 + j)\Omega$$

Elementele cuadripolului in T echivalent sunt cf. fig. 13.3.1.

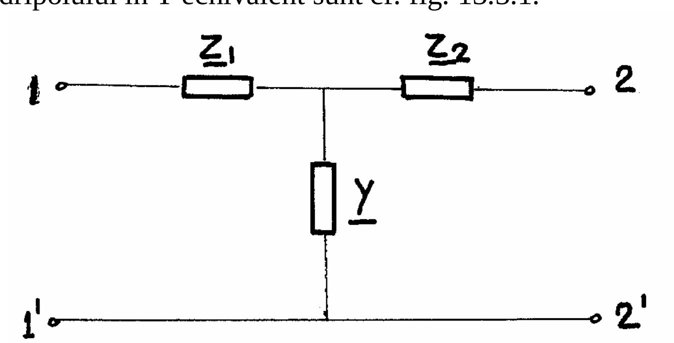

Fig. 13.3.1

$$\underline{Y} = \underline{C} = (0,5 + j \cdot 0,5)\ \text{S}$$
$$\underline{Z} = 1/\underline{Y} = 1/(0,5 + j \cdot 0,5) = (1 - j)\ \Omega$$
$$\underline{Z}_1 = (\underline{A} - 1)/\underline{C} = (-1 + j \cdot 2 - 1)/(0,5 + j \cdot 0,5) = 4j\ \Omega$$
$$\underline{Z}_2 = (\underline{D} - 1)/\underline{C} + (-0,5 + j \cdot 1,5 - 1)/(1 + j)/2 = 3j\ \Omega$$

Realizarea cuadripolului este:

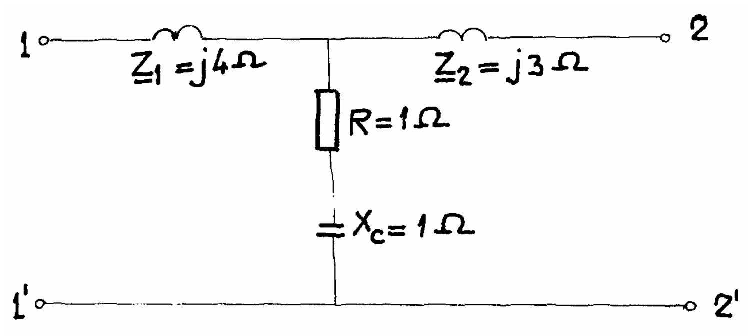

Fig. 13.3.2

13.4. (R) Sa se determine parametrii $\underline{A}, \underline{B}, \underline{C}, \underline{D}$ si $\underline{Y}_{11}, \underline{Y}_{12}, \underline{Y}_{22}$ ai cuadripolului din fig.13.4 si elementele cuadripolului in π echivalent. Date numerice: R=1Ω; X_1=2Ω; X_2=2Ω, X_M=1Ω.

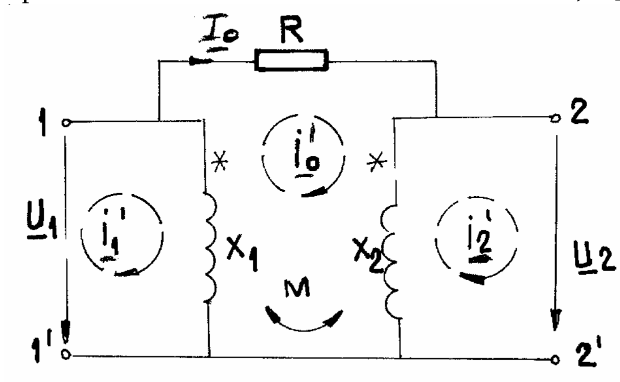

Fig. 13.4

Rezolvare: Aplicând metoda curentilor ciclici (fig.13.4), se obtine sistemul de ecuatii:

$$\underline{U}_1 = j \cdot X_1 \cdot \underline{I}'_1 - j \cdot X_M \cdot \underline{I}'_2 + j(X_M - X_1) \cdot \underline{I}'_0$$
$$-\underline{U}_2 = -j \cdot X_M \cdot \underline{I}'_1 + j \cdot X_2 \cdot \underline{I}'_2 + j(X_M - X_2) \cdot \underline{I}'_0$$
$$0 = j(X_M - X_1) \cdot \underline{I}'_1 + j(X_M - X_2) \cdot \underline{I}'_2 + [R + j(X_1 + X_2 - 2X_M)] \cdot \underline{I}'_0$$

Inlocuind datele numerice si tinând cont ca $\underline{I}_1 = \underline{I}'_1$; $\underline{I}_2 = \underline{I}'_2$; $\underline{I}_0 = \underline{I}'_0$ sistemul devine:

$$\underline{U}_1 = 2j \cdot \underline{I}_1 - j \cdot \underline{I}_2 - j \cdot \underline{I}_0$$
$$\underline{U}_2 = j \cdot \underline{I}_1 - 2j \cdot \underline{I}_2 + j \cdot \underline{I}_0$$
$$0 = -j \cdot \underline{I}_2 - j \cdot \underline{I}_1 + (1 + 2j) \cdot \underline{I}_0$$

de unde, eliminând pe $\underline{I}_0$ si explicitând curentii $\underline{I}_1$ si $\underline{I}_2$ se obtin ecuatiile:

$$\underline{I}_1 = \frac{3-2j}{3}\underline{U}_1 + \frac{-3+j}{3}\underline{U}_2$$
$$\underline{I}_2 = \frac{3-j}{3}\underline{U}_1 + \frac{-3+2j}{3}\underline{U}_2$$

Deci:

$$\underline{Y}_{11} = \frac{3-2j}{3}S;\ \underline{Y}_{12} = -\underline{Y}_{21} = \frac{-3+j}{3};\ \underline{Y}_{22} = \frac{-3+2j}{3}S$$

Pentru determinarea coeficientilor $\underline{A}, \underline{B}, \underline{C}, \underline{D}$ se expliciteaza sistemul de mai sus in raport cu $\underline{U}_1$ si $\underline{I}_1$ si se obtine:

$$\underline{U}_1 = \frac{11-3j}{10}\underline{U}_2 + \frac{9+3j}{10}\underline{I}_2$$

$$\underline{I}_1 = -\frac{1+7j}{10}\underline{U}_2 + \frac{11-3j}{10}\underline{I}_2$$

Prin urmare:

$$\underline{A} = \frac{11-3j}{10};\ \underline{B} = \frac{9+3j}{10}\Omega;\ \underline{C} = -\frac{1+7j}{10}S;\ \underline{D} = \frac{11-3j}{10}$$

Elementele cuadripolului echivalent in π se determina scriind ecuatiile care rezulta din aplicarea primei teoreme a lui Kirchhoff in nodurile M si N (fig. 13.4.1.a):

$$\underline{I}_1 = \left(\underline{U}_1 - \underline{U}_2\right)\cdot \underline{Y}_0' + \underline{U}_1 \cdot \underline{Y}_1' = \left(\underline{Y}_1' + \underline{Y}_0'\right)\cdot \underline{U}_1 - \underline{Y}_0' \cdot \underline{U}_2$$

$$\underline{I}_2 = \left(\underline{U}_1 - \underline{U}_2\right)\cdot \underline{Y}_0' - \underline{U}_2 \cdot \underline{Y}_2' = \underline{Y}_0' \cdot \underline{U}_1 - (\underline{Y}_0' + \underline{Y}_0')\cdot \underline{U}_2$$

sau:

$$\underline{Y}_{11} = \underline{Y}_1' + \underline{Y}_0';\ \underline{Y}_{22} = -\left(\underline{Y}_2' + \underline{Y}_0'\right);\ \underline{Y}_{12} = -\underline{Y}_{21} = -\underline{Y}_0'$$

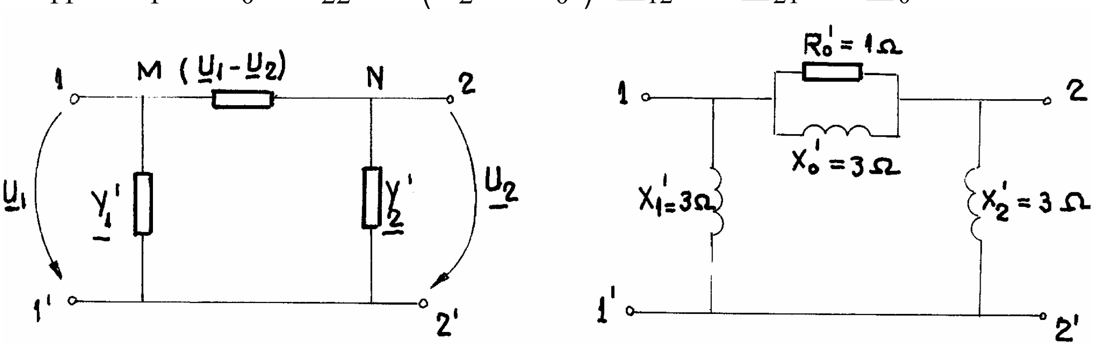

Fig. 13.4.1 a, b

de unde rezulta:

$$\underline{Y}'_1 = \underline{Y}_{11} + \underline{Y}_{12} = \frac{3-2j}{3} + \frac{-3+j}{3} = -\frac{j}{3}\ S$$

$$\underline{Y}'_2 = -\left(\underline{Y}_{22} + \underline{Y}_{21}\right) = \left(-\frac{-3+2j}{3} + \frac{3-j}{3}\right) = -\frac{j}{3}\ S$$

$$\underline{Y}'_0 = \underline{Y}_{21} = -\underline{Y}_{12} = -\frac{-3+j}{3} = \frac{3-j}{3} = \left(1 - \frac{j}{3}\right) S$$

Realizarea cuadripolului in π echivalent este indicata in fig.13.4.1.b.

13.5 (R) Sa se determine parametrii fundamentali $\underline{A}, \underline{B}, \underline{C}, \underline{D}$ ai cuadripolilor din fig.13.5.a si 13.5.b. Pentru cuadripolul din fig.13.5.b sa se determine apoi impedantele de gol si de scurtcircuit precum si coeficientii $\underline{Z}_{11}, \underline{Z}_{12}, \underline{Z}_{21}, \underline{Z}_{22}$.

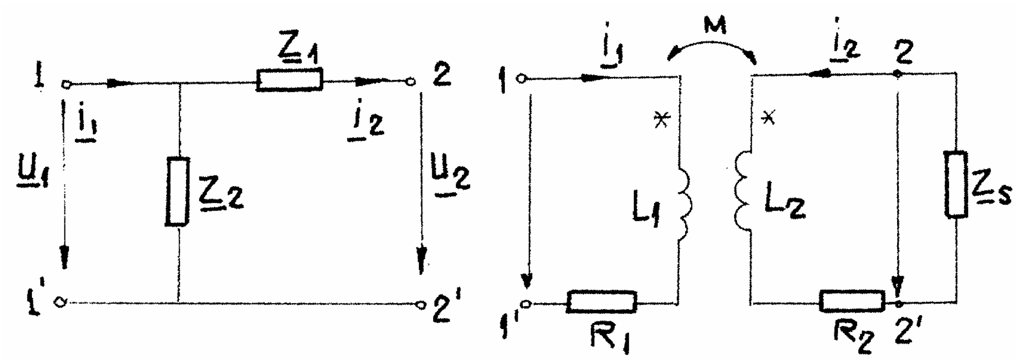

Fig. 13.5 a, b

Rezolvare: Pentru cuadripolul din fig.13.5.a prin incercarile de mers in gol si de scurtcircuit, cu alimentare pe la bornele primare se obtine:

$$\underline{I}_2 = 0;\ \underline{U}_{10} = \underline{A}\cdot\underline{U}_{20};\ \underline{I}_{10} = \underline{C}\cdot\underline{U}_{20}$$

Dar $\underline{I}_{10} = \underline{U}_{10} / \underline{Z}_2$ iar $\underline{U}_{10} = \underline{U}_{20}$

Deci: $\underline{A} = \underline{U}_{10} / \underline{U}_{20} = 1;\ \underline{C} = \underline{I}_{10} / \underline{U}_{20} = \underline{I}_{10} / \underline{U}_{10} = 1/\underline{Z}_2$

La scurtcircuitarea bornelor secundare: $\underline{U}_2 = 0$ si rezulta:

$\underline{U}_{1sc} = \underline{B}\cdot\underline{I}_{2sc};\ \underline{I}_{1sc} = \underline{D}\cdot\underline{I}_{2sc}$, iar din figura

$$\underline{I}_{2sc} = \underline{U}_{1sc} / \underline{Z}_1;\ \underline{I}_{2sc} = \frac{\underline{I}_{1sc}\cdot\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}$$

si deci: $\underline{B} = \underline{Z}_1;\ \underline{D} = \dfrac{\underline{Z}_1 + \underline{Z}_2}{\underline{Z}_2}$

In cazul cuadripolului din fig.13.5.b, aplicând teorema a II-a lui Kirchhoff in circuitul primar si in cel secundar al transformatorului se obtin ecuatiile:

$$\underline{U}_1 (R_1 + j\omega L_1)\underline{I}_1 - j\omega M\cdot\underline{I}_2$$
$$-\underline{U}_2 = -j\omega M\cdot\underline{I}_1 + (R_2 + j\omega L_2)\underline{I}_2$$

Explicitând acest sistem de ecuatii in raport cu marimile din primar, se obtin ecuatiile:

$$\underline{I}_1 = \frac{\underline{U}'_2}{j\omega M} + \frac{R_2 + j\omega L_2}{j\omega M}\cdot\underline{I}_2$$

$$\underline{U}_1 = \frac{\omega L_1 - jR_1}{\omega M}\underline{U}_2 + \left[\frac{R_1 L_2}{M} + \frac{R_2 L_1}{M} + j\left(\frac{\omega L_1 L_2}{M} - \frac{R_1 R_2}{\omega M} - \omega M\right)\right]\cdot\underline{I}_2$$ Identificând cu ecuatiile fundamentale ale cuadripolului:

$$\underline{U}_1 = \underline{A}\cdot\underline{U}_2 + \underline{B}\cdot\underline{I}_2$$
$$\underline{I}_1 = \underline{C}\cdot\underline{U}_2 + \underline{D}\cdot\underline{I}_2$$

rezulta:

$$\underline{A} = \frac{\omega L_1 - jR_1}{\omega M};\ \underline{B} = \frac{R_1 L_2}{M} + \frac{R_2 L_1}{M} + j\left(\frac{\omega L_1 L_2}{M} - \frac{R_1 R_2}{\omega M} - \omega M\right)$$

$$\underline{C} = -j\cdot 1/\omega M;\ D = \frac{R_2 + j\omega L_2}{j\omega M} = L_2 / M - j\cdot R_2 / \omega M$$

Identificând primul sistem de ecuatii cu sistemul:

$$\underline{U}_1 = \underline{Z}_{11} \cdot \underline{I}_1 + \underline{Z}_{12} \cdot \underline{I}_2$$

$$\underline{U}_2 = \underline{Z}_{21} \cdot \underline{I}_1 + \underline{Z}_{22} \cdot \underline{I}_2$$

se obtine: $\underline{Z}_{11} = R_1 + j\omega L_1$

$$\underline{Z}_{12} = -Z_{21} = -j\omega M$$

$$\underline{Z}_{22} = -(R_2 + j\omega L_2)$$

In cazul incercarii de mers in gol (I_2=0) ecuatiile devin:

$$\underline{U}_{10} = (R_1 + j\omega L_1) \cdot \underline{I}_{10}$$

$\underline{U}_{20} = j\omega M \cdot \underline{I}_{10}$, deci impedanta de gol este:

$$\underline{Z}_{10} = \underline{U}_{10} / \underline{I}_{10} = R_1 + j\omega L_1$$

Scurtcircuitând bornele secundare (U_2=0) se obtin ecuatiile

$$\underline{U}_{1sc} = (R_1 + j\omega L_1)\underline{I}_{1sc} - j\omega M \cdot \underline{I}_{2sc}$$

$$0 = -j\omega M \cdot \underline{I}_{1sc} + (R_2 + j\omega L_2) \cdot \underline{I}_{2sc}$$

de unde prin eliminarea lui $\underline{I}_{2sc}$ se obtine ecuatia:

$\underline{U}_{1sc} = \left[R_1 + j\omega L_1 + \dfrac{\omega^2 M^2}{R_2 + j\omega L_2} \right] \underline{I}_{1sc}$; impedanta de scurt-circuit, in cazul alimentarii pe la bornele primare este:

$$\underline{Z}_{1sc} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2} = j\left(\omega L_1 - \frac{\omega^2 M^2}{R_2^2 + \omega^2 L_2^2} \cdot \omega L_2 \right) +$$

$$+ R_1 + \frac{\omega^2 M^2}{R_2^2 + \omega^2 L_2^2}$$

13.6. Sa se determine parametrii fundamentali ai cuadripolului in T (gama invers) si Γ (gama) pentru $\underline{Z}_e + \underline{Z}_t$:

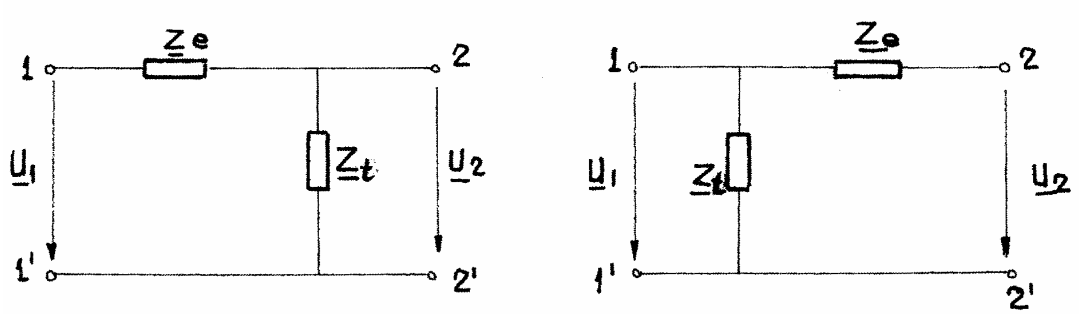

Fig. 13.6

R: a) $\underline{A} = 1 + \underline{Z}_e / \underline{Z}_t$; $\underline{B} = \underline{Z}_1$; $\underline{C} = 1/\underline{Z}_t$; $\underline{D} = 1$

b) $\underline{A} = 1$; $\underline{B} = \underline{Z}_1$; $\underline{C} = 1/\underline{Z}_t$; $\underline{D} = 1 + \underline{Z}_e / \underline{Z}_t$

13.7. Pentru cuadripolul din fig.13.7 se cunosc: $\omega L_1 = \omega L_2 = 1/\omega C = 10\Omega$, $\omega L = 5\Omega$. Se cere cuadripolul echivalent in T.

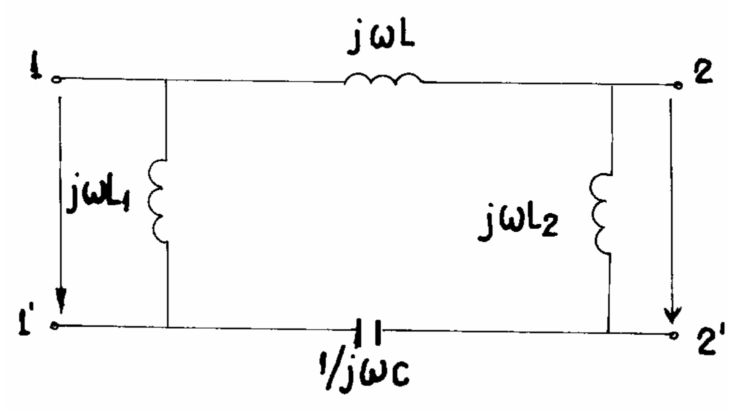

Fig. 13.7

R: $\underline{Y} = -j \cdot 0{,}15$ S; $\underline{Z}_1 = -j\ 10/3\Omega$; $\underline{Z}_2$=-j·10/3 Ω.

13.8. Se da cuadripolul din fig. 13.8 cu datele: $\omega L_3 = 1/\omega C_3 = 10\Omega$; $R_1 = R_2 = R_3 = 1\Omega$; $\omega L_1 = \omega L_2 = 2\Omega$; $1/\omega C_1 = 1\Omega$. Se cere cuadripolul echivalent in π.

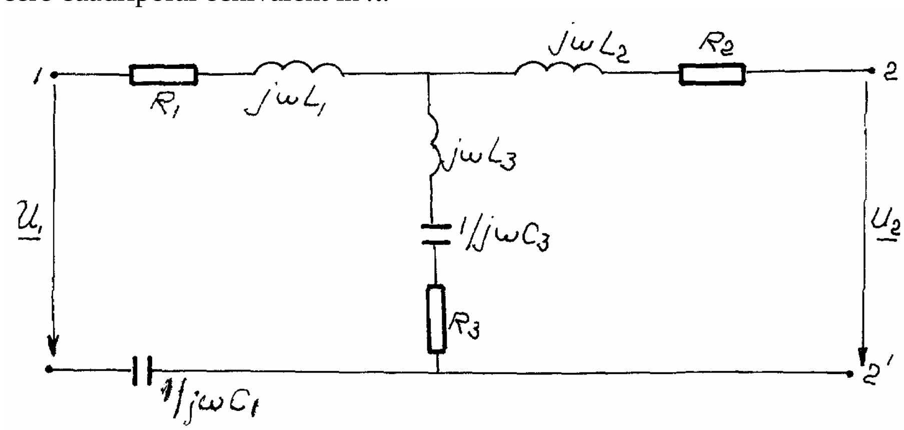

Fig.13.8

R: $\underline{Y} = 3/2 - j$; $\underline{Z}_1 = (5-j)/13$; $\underline{Z}_2 = (5-j)/26$

13.9. Se da cuadripolul din fig.13.9 cu datele: $\omega L_1 = 1/\omega C = 4\Omega$, $\omega L_2 = \omega L_2' = \omega L = 2\Omega$. Se cere cuadripolul echivalent in T.

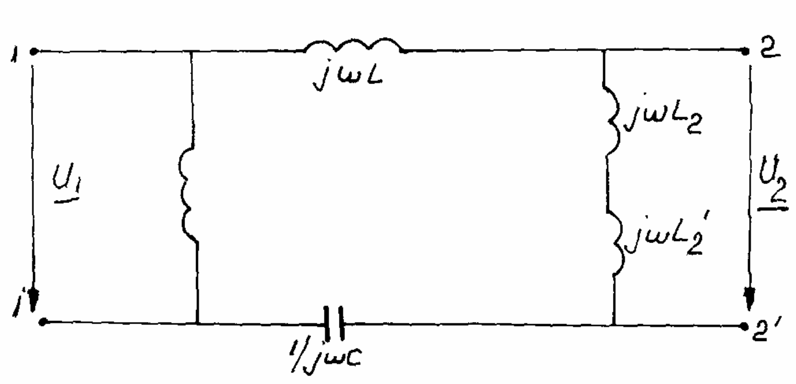

Fig. 13.9

R: $\underline{Y} = -3/8 \cdot j$; $\underline{Y}_1 = -4/3 \cdot j$; $\underline{Z}_2 = -4/3 \cdot j$

13.10. Pentru cuadripolul din fig. 13.10 se cunosc: $R_1=R_2=R_3=1\ \Omega$; $\omega L_1=4\ \Omega$; $\omega L_1'=8\ \Omega$; $\omega L_2=\omega L_3=\omega L_{23}=6\ \Omega$; $1/\omega C_2=10\ \Omega$; $1/\omega C_3=12\ \Omega$; $\omega L_{11}=3\ \Omega$. Se cer cuadripolii echivalenti in T si π.

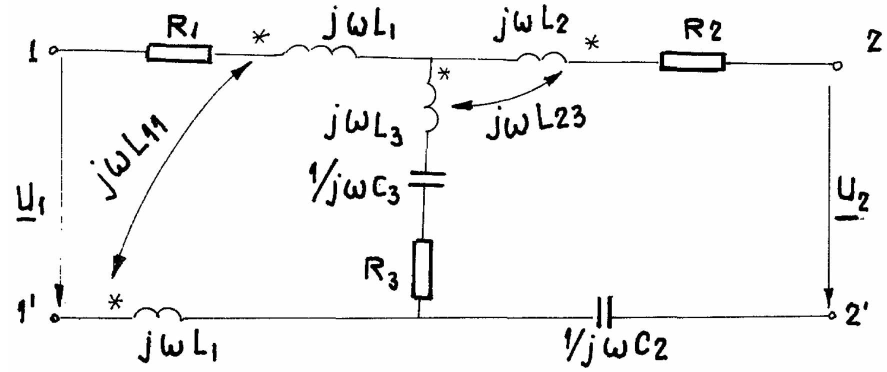

Fig. 13.10

R: a) $\underline{Y}=1$ S; $\underline{Z}_1=1\Omega$; $\underline{Z}_2=(1+2j)\ \Omega$

b) $\underline{Z}=3+4j\Omega$; $\underline{Y}_1=(11+2j)25\cdot S$; $\underline{Y}_2=(3-4j)25\cdot S$