CAP.6.

B) CIRCUITE MONOFAZATE IN REGIM SINUSOIDAL

6.6. (R) Pentru a se determina inductivitatea L si rezistenta R a unei bobine se masoara valoarea efectiva a tensiunii U la bornele bobinei si a curentului care o strabate, la doua frecvente diferite f_1 si f_2 ale tensiunii sinusoidale aplicate si anume:

U_1=60 V; I_1=10 A; f_1=50 Hz

U_2=60 V; I_2=6 A; f_2=100 Hz

Sa se calculeze inductivitatea si rezistenta R a bobinei.

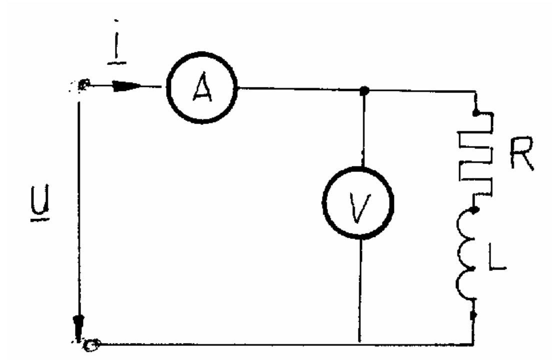

Fig.6.6

Rezolvare: Modulul impedantei complexe echivalente a bobinei este in cele doua cazuri:

$$\frac{U_1}{I_1} = \sqrt{R^2 + \omega_1^2 \cdot L^2}; \omega_1 = 2 \cdot \pi f_1$$

$$\frac{U_2}{I_2} = \sqrt{R^2 + \omega_2^2 \cdot L^2}$$

Se obtine sistemul $\begin{cases} R^2 + 10^4 \cdot \pi^2 \cdot L^2 = 36 \\ R^2 + 4 \cdot 10^4 \cdot \pi^2 \cdot L^2 = 100 \end{cases} \Rightarrow \begin{matrix} L = 14{,}7 \cdot 10^{-3} H \\ R = 3{,}84 \Omega \end{matrix}$

6.7. (R) Un rezistor cu rezistenta R=100 Ω si un condensator de capacitate C=2μF sunt conectate in serie. Tensiunea la bornele condensatorului are valoarea instantanee:

$u_c = 10 \cdot \sin 5000\, t$

Sa se calculeze tensiunea la bornele rezistorului u_R si la bornele intregului circuit u, precum si puterea instantanee absorbita.

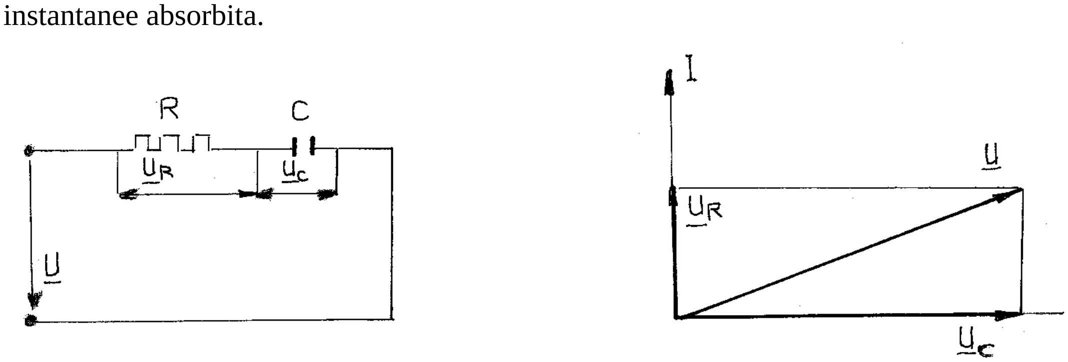

Fig. 6.7

Rezolvare: u_c=10·sin 5000 t

Reprezentarea in complex simplificat a tensiunii u_c este:

$$\underline{U}_c = 10/\sqrt{2} \cdot e^{j0}$$

Impedanta complexa este:

$$\underline{Z}_c = \frac{1}{j\omega C} = 10^2 \cdot j = 10^2 \cdot e^{-j\cdot\pi/2}$$

Curentul care strabate circuitul este in complex:

$$\underline{I} = \frac{\underline{U}_c}{\underline{Z}_c} = \frac{10 \cdot e^{j0}}{\sqrt{2} \cdot 10^2 \cdot e^{-j\pi/2}} = \frac{1}{10 \cdot \sqrt{2}} \cdot e^{j\pi/2}$$

iar in valoare instantanee:

$$i = \sqrt{2} \cdot \frac{1}{10 \cdot \sqrt{2}} \cdot \sin(\omega t + \pi/2)$$

Tensiunea la bornele intregului circuit este:

$$\underline{U} = \underline{U}_R + \underline{U}_c = j \cdot 10/\sqrt{2} + 10/\sqrt{2} = 10/\sqrt{2}(i + j) = 10 \cdot e^{j\pi/4}$$

si deci:

$$u = \sqrt{2} \cdot 10 \cdot \sin(\omega t + \pi/4)$$

Puterea instantanee absorbita de circuit este:

$$p = u \cdot i = \sqrt{2} \cdot 10 \cdot \sin(\omega t + \pi/4) \cdot [1/10 \cdot \sin(\omega t + \pi/2)]$$

Cum $\sin a \cdot \sin b = \frac{1}{2}[\cos(a - b) - \cos(a + b)]$, rezulta:

$$p = \frac{\sqrt{2}}{2} \cdot \left[\frac{\sqrt{2}}{2} + \sin\left(2\omega t + \frac{\pi}{4}\right)\right] = 0,5 + 0,707 \cdot \sin\left(10^4 t + \frac{\pi}{4}\right)$$

6.8. (R) O bobina de parametrii R=10 Ω si L=10 mH conectata in serie cu un condensator de capacitate C=50 μF este parcursa de curentul i=0,2 sin (1000t).

Sa se calculeze valorile instantanee ale tensiunilor u, u_R, u_L, u_C si sa se efectueze bilantul puterilor.

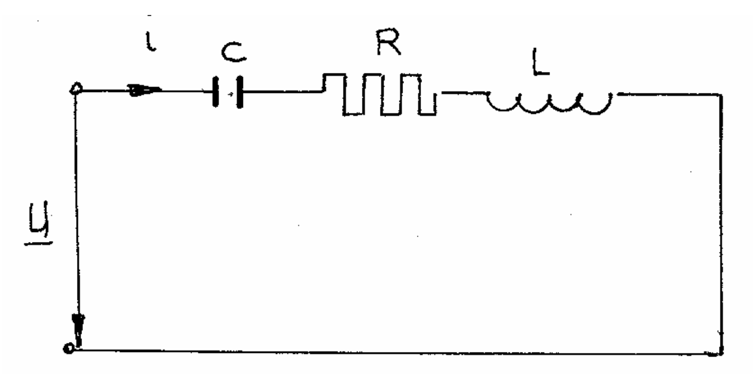

Fig. 6.8

Rezolvare: Curentul care strabate circuitul este:

$i = I_M \sin\omega t = \sqrt{2} \cdot I \cdot \sin\omega t$, unde:

$$I_M = 0{,}2A; I = \frac{0{,}2}{\sqrt{2}} = 0{,}142A; \omega = 1000\ rad/s$$

Reprezentarea in complex simplificat a curentului este:
I=0.142 $e^{j\cdot o}$
Reactantele inductiva si capacitiva ale circuitului sunt:

$$X_L = \omega L = 10\Omega; X_C = -\frac{U}{\omega C} = -20\Omega$$

Impedanta complexa a circuitului este:

$$\underline{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right) = 10 - j\cdot 10 = 10\cdot\sqrt{2}\cdot e^{-j\cdot\pi/4}$$

Tensiunile sunt:

$$\underline{U} = \underline{Z}\cdot\underline{I} = 2\cdot e^{-j\cdot\pi/4} = \sqrt{2}(1-j)$$

$$\underline{U}_R = R\cdot\underline{I} = 10\cdot 0{,}142\cdot e^{j\cdot o} = 1{,}42$$

$$\underline{U}_L = j\cdot\omega L\cdot\underline{I} = 1{,}42\cdot e^{j\pi/2} = j\cdot 1{,}42$$

$$\underline{U}_C = \frac{1}{j\omega C}\cdot\underline{I} = 2{,}84\cdot e^{-j\cdot\pi/2} = -j\cdot 2{,}84$$

Verificare:

$$\underline{U} = \underline{U}_R + \underline{U}_L + \underline{U}_C = \sqrt{2}(1-j)$$

Valorile instantanee ale tensiunilor sunt:

$$u = \sqrt{2}\cdot 2\cdot\sin(\omega t - \pi/4)$$

$$u_R = \sqrt{2}\cdot 1{,}42\cdot\sin\omega t$$

$$u_L = \sqrt{2}\cdot 1{,}42\cdot\sin(\omega t + \pi/2)$$

$$u_C = \sqrt{2}\cdot 2{,}84\cdot\sin(\omega t - \pi/2)$$

Puterea aparenta complexa este:

$$\underline{S} = \underline{U}\cdot\underline{I}^* = 0{,}284\cdot e^{-j\cdot\pi/4} = (0{,}2 - j\cdot 0{,}2)VA$$

$$\underline{S} = P + j\cdot Q$$

$P=R\cdot I^2=10\cdot 0{,}142^2=0{,}2$ W
$Q=X_L\cdot I^2+X_C\cdot I^2=-0{,}2$ Var
Bilantul puterilor este verificat.

6.9. (R) La variatii ale parametrilor receptorului R_1, X_1 nu se poate determina semnul unghiului φ_1 numai cu aparatele din fig.6.9 (ampermetru, voltmetru si wattmetru). In acest scop se recurge la o masuratoare suplimentara cu o bobina conectata in serie. Aparatele conectate in circuit indica:

1. Intrerupatorul k inchis: U=100 V; I=10 A; P=300 W.
2. Intrerupatorul k deschis: U=100 V; I=12,5 A; P=625 W.

Sa se calculeze R_1, X_1, Z_1 ale receptorului si R_2, X_2, Z_2 ale bobinei introduse.

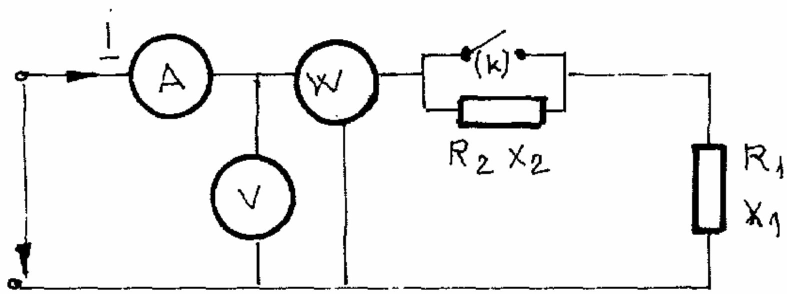

Fig. 6.9

Rezolvare:

1. Intrerupatorul k inchis:

Impedanta complexa a circuitului are modulul:

$$Z = \frac{U}{I} = \frac{100}{10} = 10\Omega$$

si argumentul:

$$\varphi = \arccos\frac{P}{U \cdot I} = \arccos(0{,}3) = 72^0 30'$$

Deci: $\underline{Z}_1 = 10 \cdot e^{\pm j \cdot 72^0 30' \cdot \pi : 180} = 3 \pm j \cdot 9{,}55$

2. Intrerupatorul k deschis:

$$Z_1 = \frac{U}{I} = \frac{100}{12{,}5} = 8\Omega$$

$$\varphi = \arccos\frac{P}{U \cdot I} = \arccos(0{,}5) = 60^0 20'$$

$$\underline{Z} = 8 \cdot e^{j \cdot 60^0 20' \pi : 180^0} = 4 + j \cdot 6{,}96$$

Impedanta complexa echivalenta este:

$\underline{Z} = \underline{Z}_1 + \underline{Z}_2$ de unde:

$$\underline{Z}_2 = \underline{Z} - \underline{Z}_1 = 4 + j \cdot 6{,}96 - 3 \pm j \cdot 9{,}5 = \begin{cases} 1 - j \cdot 2{,}59 \\ 1 + j \cdot 16{,}51 \end{cases}$$

Se stie ca receptorul $\underline{Z}_2$ este inductiv, deci:

$$\underline{Z}_2 = 1 + j \cdot 16{,}51$$

ceea ce conduce la concluzia ca receptorul $\underline{Z}_1$ este capacitiv:

$$\underline{Z}_1 = 3 - j \cdot 9{,}55 = 10 \cdot e^{-j \cdot 72^0 30' \cdot \pi : 180^0}$$

Schema circuitului devine:

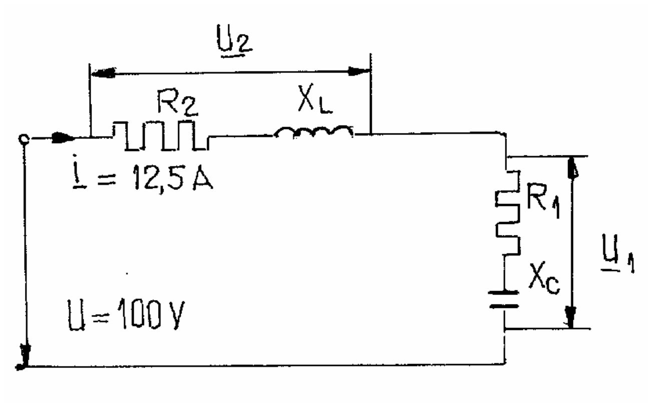

Fig. 6.9.1

6.10 (R) Pentru circuitul din fig.6.10 se cunosc: $e_1(t)=18\sqrt{2}\cdot\sin\omega t$, $e_2(t)=12\cdot\sqrt{2}\cdot\cos(\omega t-\pi)$; $R_1=3\Omega$; $\omega L_1=4\Omega$; $\omega L_1'=1\Omega$; $\frac{1}{\omega C_1}=5\Omega$; $R_2=2\Omega$; $\omega L_2=4\Omega$; $\frac{1}{\omega C_2}=5\Omega$; $\omega L_3=3\Omega$; $\frac{1}{\omega C_3}=4\Omega$; $\omega L_{11}=\omega L_{23}=1\Omega$.

Sa se determine valorile instantanee ale curentilor din laturi si sa se verifice bilantul puterilor.

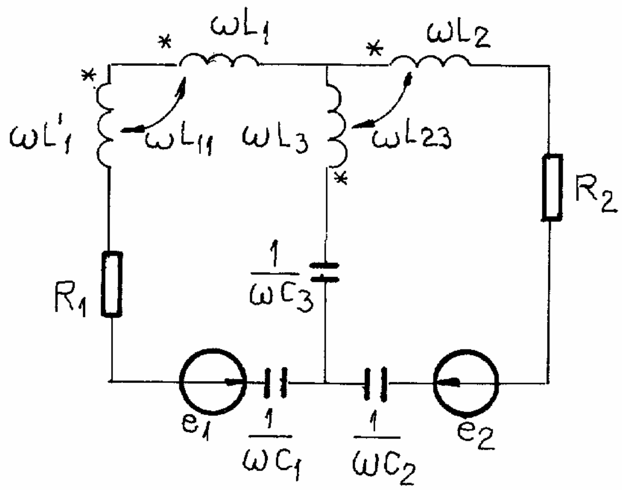

Fig. 6.10

Rezolvare:

Se aplica metoda teoremelor lui Kirchhoff sub forma complexa directa. Cu:

$$e_1(t)=18\cdot\sqrt{2}\cdot\sin\omega t \Leftrightarrow \underline{E}=18\cdot e^{j0}=18 \text{ V}$$

$$e_2(t)=12\cdot\sqrt{2}\cdot\cos(\omega t-\pi)=12\cdot\sqrt{2}\cdot\sin(\omega t-\pi/2)\Leftrightarrow$$

$\underline{E}_2=12\cdot e^{-j\cdot\pi/2}=-j\cdot 12$ V se poate realiza urmatoarea schema in complex:

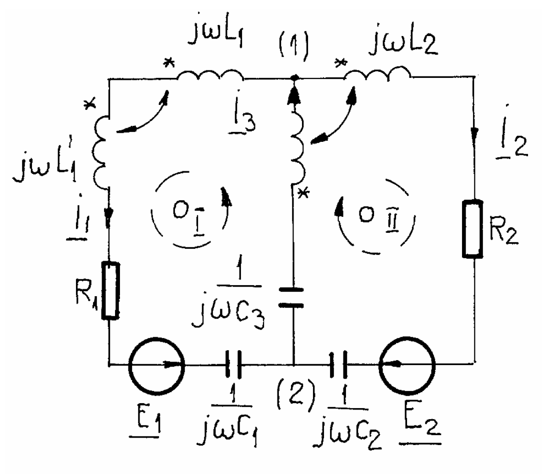

Fig. 6.10.1

Cu sensurile adoptate in figura 6.10.1 si considerând nodul (2) de referinta, rezulta:

$$-\underline{I}_1-\underline{I}_2+\underline{I}_3=0$$

$$\left[R_1+j\left(\omega L_1+\omega L_1{}'-\frac{1}{\omega C_1}\right)\cdot\underline{I}_1-2j\omega L_{11}\cdot\underline{I}_1+j\left(\omega L_3-\frac{1}{\omega C_3}\right)\right]\cdot$$

$$\cdot\underline{I}_3+j\omega L_{23}\cdot\underline{I}_2=\underline{E}_1$$

$$R_2+j\left(\omega L_2-\frac{1}{\omega C_2}\right)\cdot\underline{I}_2+j\left(\omega L_3-\frac{1}{\omega C_3}\right)\cdot\underline{I}_3+$$

$$+j\cdot\omega L_{23}\cdot\underline{I}_3+j\cdot\omega L_{23}\underline{I}_2=\underline{E}_2$$

Sau numeric:

$$\underline{I}_3=\underline{I}_1+\underline{I}_2$$

$$(3-j\cdot 2)\underline{I}_1+j\cdot\underline{I}_2-j\cdot\underline{I}_3=18$$

$2\cdot\underline{I}_2=-j\cdot\underline{I}_2$

cu solutiile: $\underline{I}_1=3(1+j)=3\sqrt{2}\cdot e^{j\pi/4}A$; $\underline{I}_2=-j\cdot 6=6\cdot e^{-j\cdot\pi/2}A$ si $\underline{I}_3=3(1-j)=3\cdot\sqrt{2}\cdot e^{-j\pi/4}A$, carora le corespund:

$i_1(t)=6\cdot\sin(\omega t+\pi/4)A$; $i_2(t)=6\cdot\sqrt{2}\cdot\sin(\omega t-\pi/2)A$;

$i_3(t)=6\cdot\sin(\omega t-\pi/4)A$

Aplicând ecuatia de bilant a puterilor se obtine:

$\underline{S}_g=\underline{E}_1\cdot\underline{I}_1^*+\underline{E}_2\cdot\underline{I}_2^*=18\cdot 3(1-j)+(-j\cdot 12)\cdot j\cdot 6=126-j\cdot 54$, adica:

P_g=126 W; Q_g=-54 VAr, respectiv:

$$P_R+j\cdot Q_X=R_1\cdot I_1^2+R_2\cdot I_2^2+j\left[\left(\omega L_1+\omega L_1'-\frac{1}{\omega C_1}\right)\cdot I_1^2+\right.$$

$$+\left(\omega L_2-\frac{1}{\omega C_2}\right)\cdot I_2^2+\left(\omega L_3-\frac{1}{\omega C_3}\right)\cdot I_3^2-2\,L_{11}\cdot I_1\cdot I_1\cdot\cos(\gamma_1-\gamma_1)+$$

$\left.+2\omega L_{23}\cdot I_2\cdot I_3\cdot\cos(\gamma_2-\gamma_3)\right]$=126-j·54, adica:

P_R=126 W; Q_X=-54 VAr
Cum $P_g=P_R$, $Q_g=Q_X$, arata ca rezultatele obtinute sunt corecte.

6.11 (R) Pentru circuitul din fig.6.11 se cunosc:

$\underline{E}_1 = 4 \cdot \sqrt{2} \cdot e^{j \cdot \pi / 4} = 4(1+j)$V; $\underline{I}_g = 2 \cdot e^{j^0} = 2$ A; $R_1=2\Omega$; $\omega L_1 = 6\Omega$; $\frac{1}{\omega C_1} = 10\Omega$; $\omega L_2 = 8\Omega$; $\frac{1}{\omega C_2} = 6\Omega$; $\omega L_{12} = 2\Omega$. Se cer valorile instantanee ale curentilor din laturi si sa se verifice bilantul puterilor.

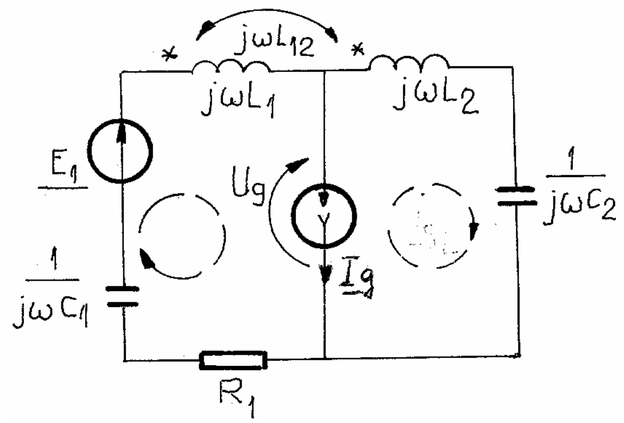

Fig. 6.11

Rezolvare: Se aplica metoda teoremelor lui Kirchhoff sub forma complexa directa. Cu sensurile adoptate in fig.6.11, rezulta:

$$\underline{I}_1 - \underline{I}_2 = \underline{I}_g$$

$$\left(R_1 + j\omega L_1 + \frac{1}{j\omega C_1}\right)\underline{I}_1 + j\omega L_{12} \cdot \underline{I}_2 - \underline{U}_g = \underline{E}_1$$

$$j\omega L_{12}\underline{I}_1 + \left(j\omega L_2 + \frac{1}{j\omega C_2}\right) \cdot \underline{I}_2 + \underline{U}_g = 0$$

sau numeric:

$$\begin{cases} \underline{I}_1 - \underline{I}_2 = 2 \\ (2 - j \cdot 4) \cdot \underline{I}_1 + j \cdot 2 \cdot \underline{I}_2 - \underline{U}_g = 4(1+j) \\ j \cdot 2 \cdot \underline{I}_1 + j \cdot 2 \cdot \underline{I}_2 + \underline{U}_g = 0 \end{cases}$$

cu solutiile: $\underline{I}_1 = 2(2+j) = 2 \cdot \sqrt{2} \cdot e^{j \cdot \text{arctg} 1/2}$ A;

$\underline{I}_2 = 2(1+j) = 2 \cdot \sqrt{2} \cdot e^{j\pi/4}$ A;

$\underline{U}_g = 4(2 - j \cdot 3) = 4 \cdot \sqrt{130} \cdot e^{-j \cdot \text{arctg} 3/2}$ V, carora le corespund:

$i_1(t) = 2 \cdot \sqrt{10} \cdot \sin\left(\omega t + 26^0 34' \cdot \pi / 180\right)$ A;

$i_2(t) = 4 \cdot \sin(\omega t + \pi/4)$ A;

$$u_g(t) = 4 \cdot \sqrt{26} \cdot \sin\left(\omega t - 56^0 10' \cdot \pi / 180\right) \text{ V.}$$

Verificarea bilantului puterilor:

$$\underline{S}_g = \underline{E}_1 \cdot \underline{I}_1^* + \underline{U}_g \cdot \underline{I}_g^* = 4(1 + j) \cdot (2 - j) + 4 \cdot (2 - j \cdot 3) \cdot 2 = 40 - j \cdot 16$$

adica: Pg=40W; Qg=-16VAr.

$$P_R + j \cdot Q_X = R_1 \cdot I_1^2 + j\left(\omega L_1 - \frac{1}{\omega C_1}\right) \cdot I_1^2 + \left(\omega L_2 - \frac{1}{\omega C_2}\right) \cdot I_2^2 +$$

$\omega L_{12}\left(\underline{I}_1 \cdot \underline{I}_2^* + \underline{I}_2 \cdot \underline{I}_1^*\right) = 40 - j \cdot 16$, adica: P_R=40 W; Q_X=-16 VAr deci rezultatele obtinute sunt corecte.