Politecnico di Torino
Dipartimento di Ingegneria Elettrica

DISTRIBUTION SYSTEM ANALYSIS

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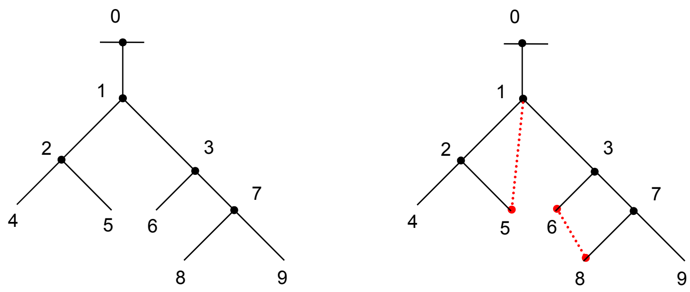
Outline

- Structure of the *Medium Voltage* distribution systems
- Classification of the *users*
- *Model* of the distribution system components
- Methods of *analysis* of the distribution systems
- The *backward/forward sweep* method
- Application *examples*

Structure of the distribution systems

- The Medium Voltage distribution system:
  - has a *weakly meshed* structure
  - is operated with radial *configurations* in order to simplify the protection schemes
  - the radial configuration is formed by *opening* the redundant branches



Radial configuration

- The choice of the branches to open can be made by using different criteria, the most used are
  - *loss* minimisation
  - *operation cost* minimisation
  - optimisation of specific *reliability* indicators
- Distribution system *supply*:
  - the distribution system can be supplied from *multiple points*
  - traditionally, the supply points are the HV/MV transformation *substations*
  - the development of *local* generation systems has increased the number of supply points in the distribution system
  - at the operation level, the system *radial* configuration concerns the *portion* of the system supplied by the same HV/MV substation
  - additional sources in the radial configurations make protection schemes and procedures power flow calculation more *complicated*

Medium Voltage system

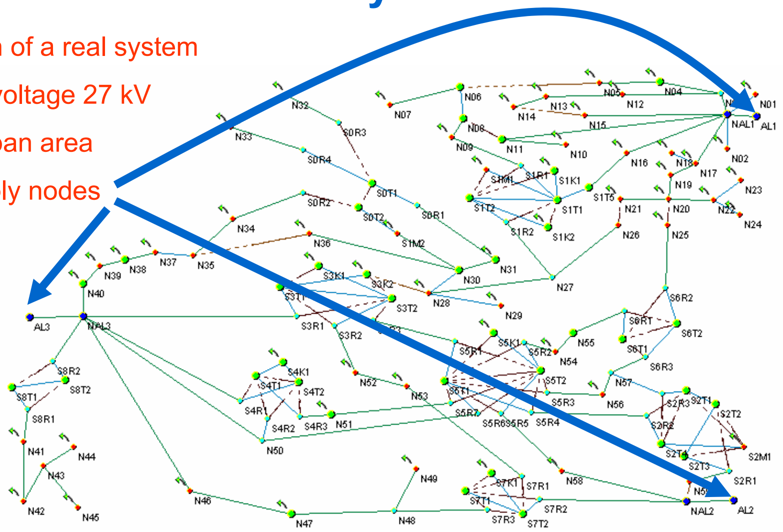
- Types of *nodes*:
  - supply with protection
  - MV/MV substation with disconnects
  - loads (MV users and MV/LV substations)
- Degree of *automation* of the nodes:
  - *rigid* nodes (no accessible switching device)
  - *remote-controlled* nodes (*automatic* switching from remote centre)
  - *locally-controlled* nodes (by local intervention of the operator teams)
- System *branches*:
  - overhead or cable *lines*, *transformers*
  - the branches not connected to supply nodes have no circuit breaker, but only switches at the two sides
  - in the MV system with isolated neutral, the switch at the side with the *lower degree of automation* of each open branch is kept *closed* in order to simplify the switching operations

Urban and extraurban MV systems

- Characteristics:
  - the *urban* distribution systems are mainly formed by cable lines
  - the *extraurban* distribution systems mainly contain *overhead* lines
  - *load density* is a key difference between urban/extraurban systems
- *Load density*:
  - is represented by the *distance of action* of the substations as characteristic parameter
  - the *distance of action* indicates the *length* of the lines starting from the substations
  - *urban centres*: distance of action of about one km
  - *rural areas*: distance of action of one order of magnitude higher
  - the choice of the distance of action depends on the *trade-off* between installation and operation costs
  - low distance of action corresponds to a large number of substations installed, but low losses for the single substation (due to the lower line length), and viceversa

MV Test System

- portion of a real system
- rated voltage 27 kV
- suburban area
- 3 supply nodes



Node classification

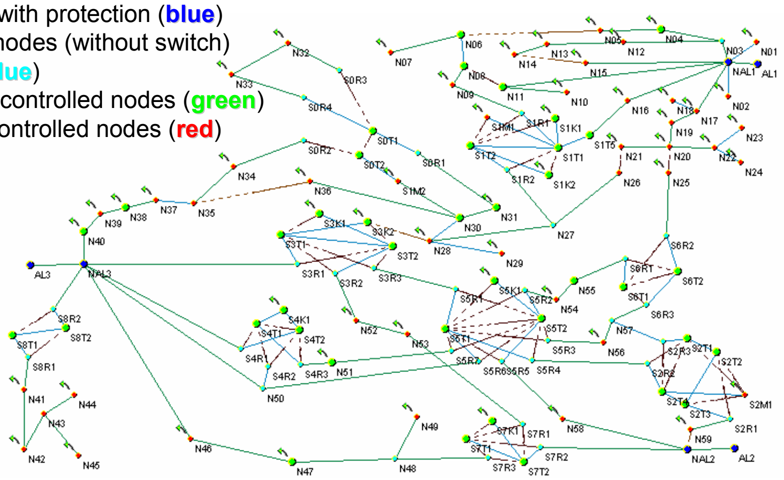
123 load nodes

6 nodes with protection (**blue**)

35 "rigid" nodes (without switch) (**light blue**)

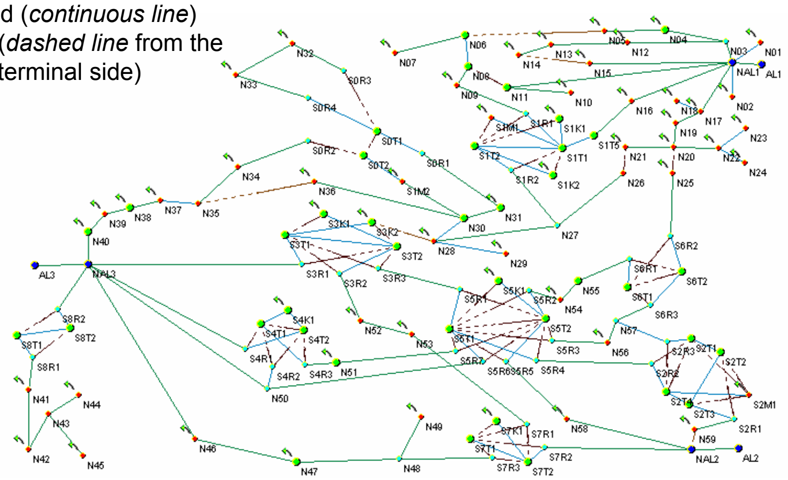
39 remote-controlled nodes (**green**)

46 locally-controlled nodes (**red**)



Branch classification

180 branches
123 closed (*continuous line*)
57 open(*dashed line* from the *open* terminal side)

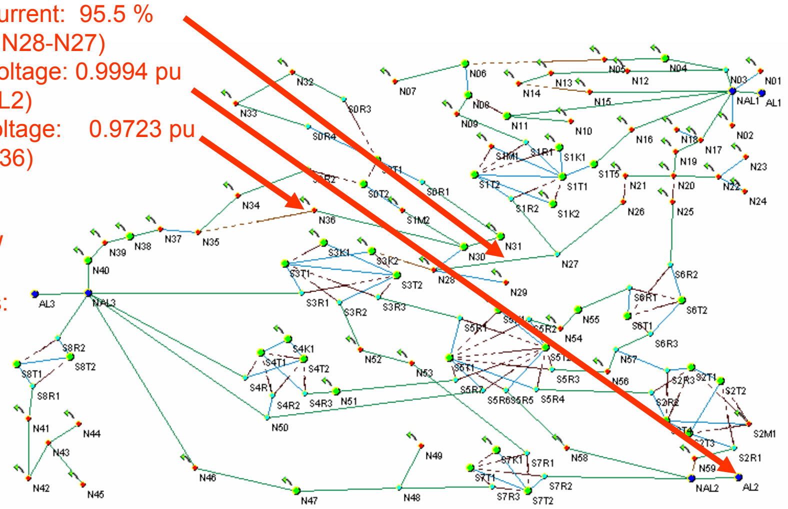


Electrical calculations in the base case

Maximum current: 95.5 %
(branch N28-N27)
Maximum voltage: 0.9994 pu
(node AL2)
Minimum voltage: 0.9723 pu
(node N36)

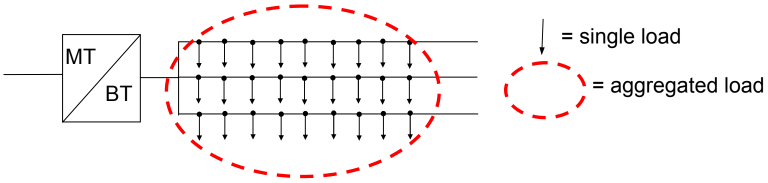
Total load:
72.92 MW

Total losses:
1.015 %



Classification of the users

- Classification based on the *energy use*:
  - *residential* users
  - *industrial* users
  - users of the *tertiary sector*
  - *other* users (e.g., lighting, traction, etc.)
- Each user may exhibit a *variable* load pattern, depending on the type of use of the energy
- In several cases the distribution system does not supply each residential user individually, but supplies an *aggregated load*

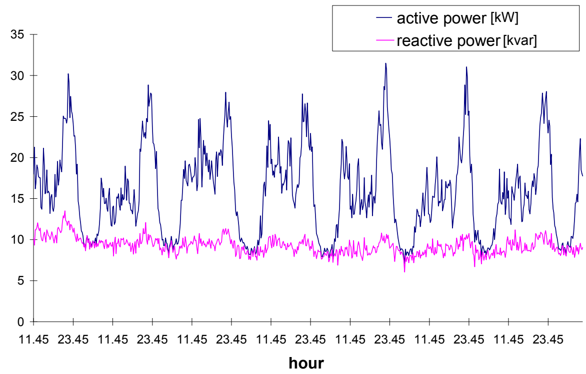


Load aggregation

- For a *residential area*:
  - the consumption may vary in function of the *number* of persons in the family, of the *activity* of the persons and of their *lifestyle*
  - the *characterization* of the residential consumption by taking into account the possible load pattern of the electrical appliances would require a *statistical analysis* based on the various aspects affecting the energy use in the family
  - fortunately, the aggregated load pattern for a *significant number* of residential customers (e.g., 20-100) connected to the same feeder or substation can be forecast in a relatively easy way
  - the different behavior of the single customers (families) leads to an *overall daily evolution* of the total load with some regularities
- *Other* users:
  - large industrial and tertiary users are supplied *individually*
  - It is possible to define the *load patterns* for the single loads

Aggregated residential load

Composition	Number of users	reference power [kW]
Residential load	80	237.5
General services of the buildings	8	50
Other	--	--



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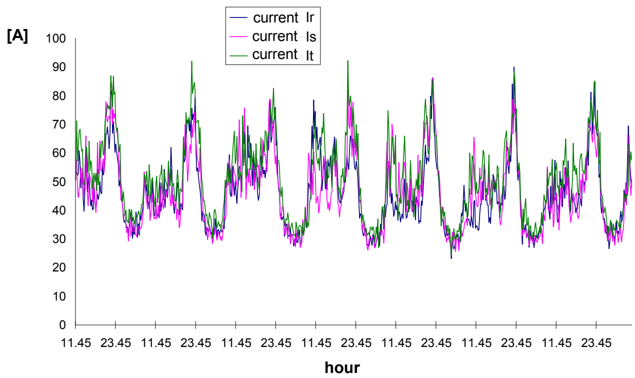
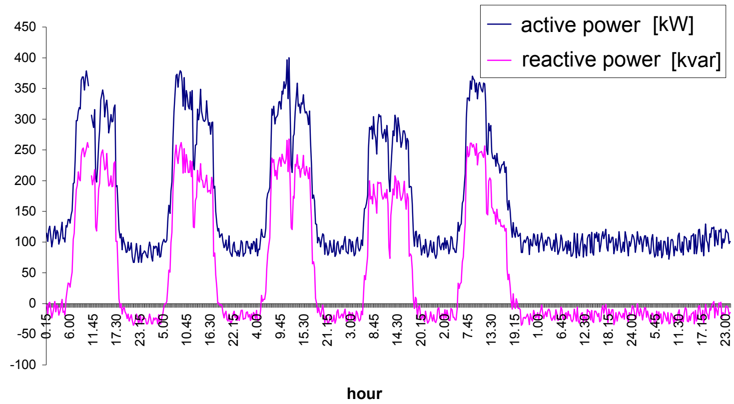


Industrial load

reference power [kW] 400

rated voltage [kV] 6.3

utilization medium

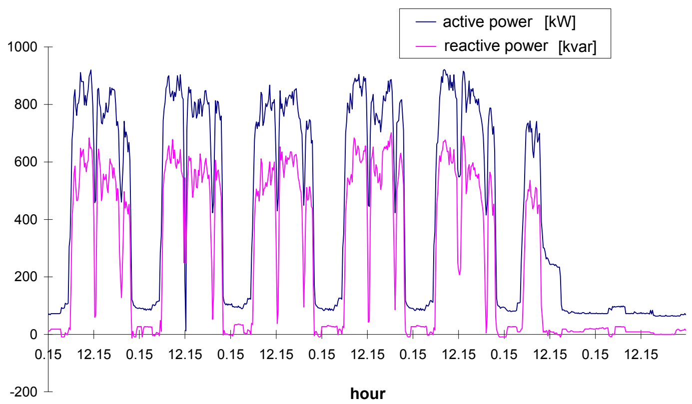


Industrial load

reference power [kW] 1000

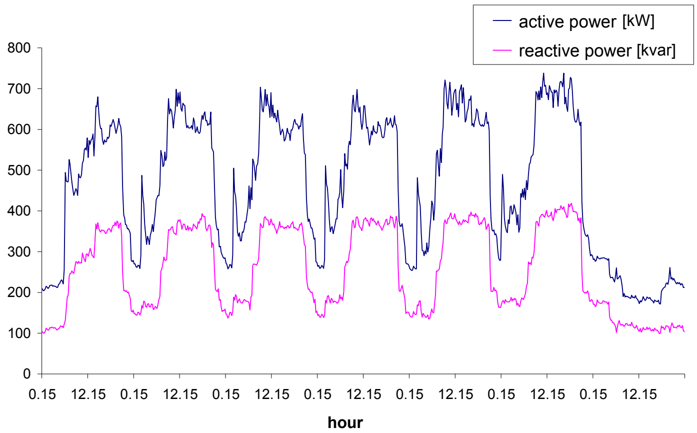
rated voltage [kV] 27

utilization high



Consumer of the tertiary sector

reference power [kW] 800 | **rated voltage [kV]** 6.3 | **utilization** high



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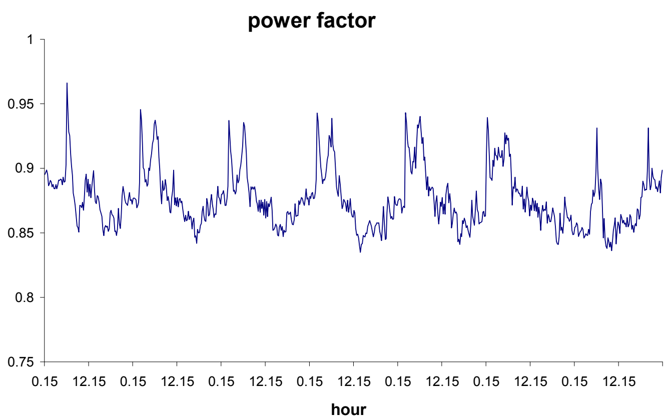
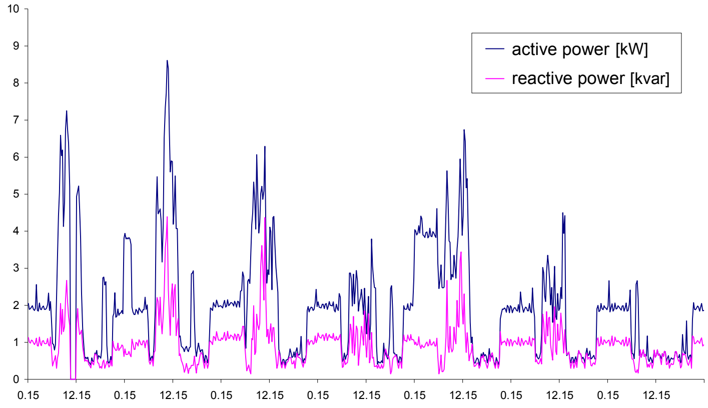


Consumer of the tertiary sector

reference power [kW] 15

rated voltage [V] 400

utilization medium

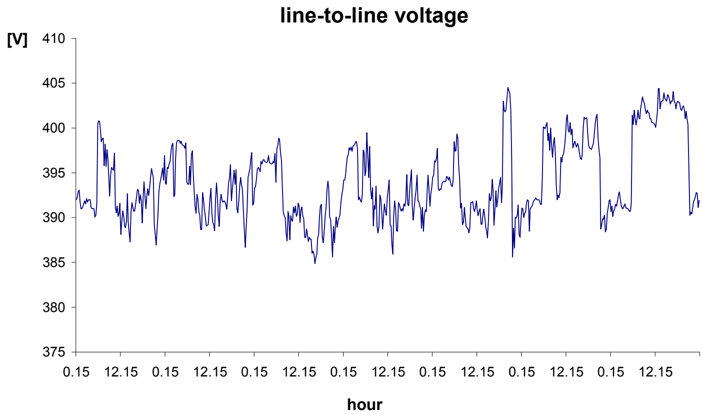


Consumer of the tertiary sector

reference power [kW] 15

rated voltage [V] 400

utilization medium



Load patterns

- *Residential users*:
  - Load pattern with significant portion of *base power* due to the *diversity* among the aggregation of similar loads (e.g., refrigerators) although each of them has cycling (intermittent) operation
  - higher consumption during the day (with concentration of the activities) and lower (but non-zero) at night
- *Industrial users*:
  - typical patterns with two peaks due to the working activity in the morning and in the afternoon and to the lunch pause
  - energy request reduced during the night
- *Tertiary users*:
  - *medium-small users* (e.g., small commercial activities and offices): load profile similar to the industrial one
  - *large users* (e.g., shopping malls and large offices): single peak during the day due to continuing working period, and non-negligible demand at night, with services in continuous operation (e.g., refrigerators and lighting)

Load profiles

- After the introduction of the competitive electricity market, the energy suppliers may *new degrees of freedom* to formulate new tariff structures
- The knowledge of the *electrical load evolution* is essential for the definition of the time-variable tariffs
- From detailed analysis carried out on specific load categories, the load patterns representative of load aggregations (*load profiles*) are extracted
- The load profiles are *normalized* with respect to the peak of the load pattern, to facilitate their use with different load aggregations
- The load profiles are used to *forecast* the evolution of the consumption at the HV/MV or MV/LV substation level
- This information allow for identifying *criticality* and *periodicity* (weekly, monthly or seasonal) of the consumption oscillations

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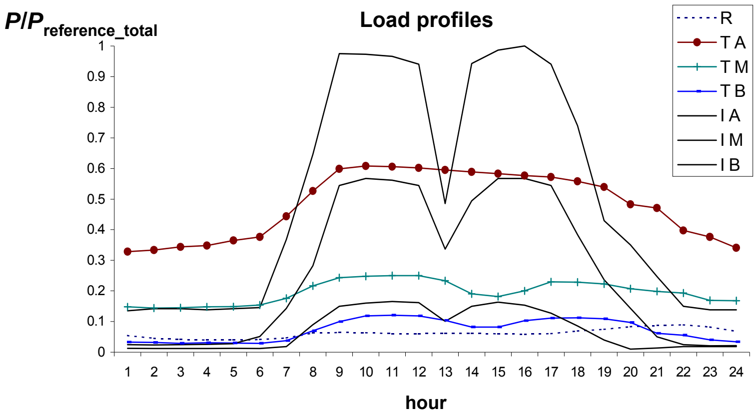
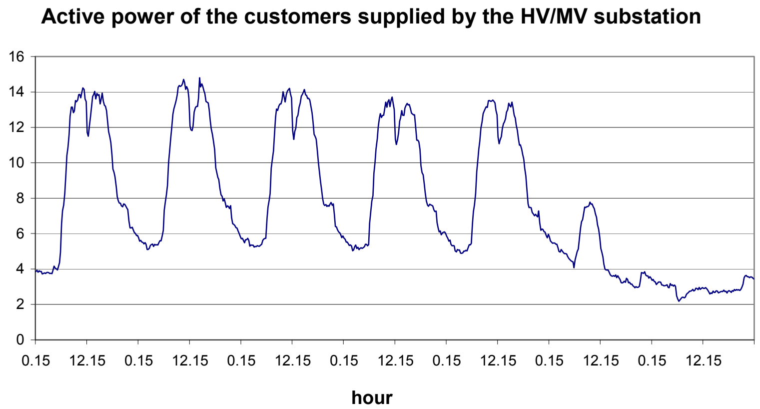
Normalized load profiles

R = residential
I = industrial
T = tertiary

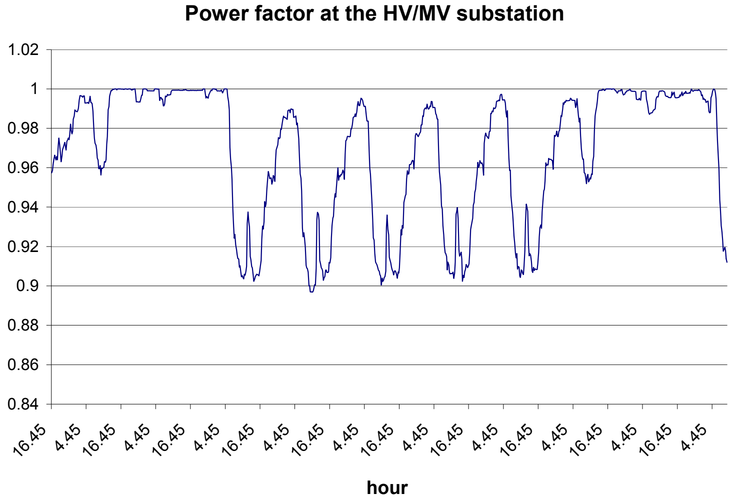
A = high utilization
M = medium utilization
B = low utilization



HV/MV substation



HV/MV substation



Objectives of the system analysis

- The *analysis* of the distribution network requires:
  - the knowledge of the *network structure*: the topology may vary during the time, with possible branches not used for maintenance or faults, substituted by the redundant branches to maintain the radial configuration
  - the knowledge of the *loads connected*: the loads may be very different among them both for their electrical nature and for their supply parameters
  - "know" the loads means the availability of the *evolution* of the active and reactive power for all the aggregated users supplied by the network under analysis
  - often the power factor is assumed *constant* and the evolution of the reactive power exchanged is estimated
  - the hypothesis of constant power factor is often plausible: below a certain limit the payment of *extra fees* is required
  - the users apply load *compensation*, so that the power factor may be reasonably (under-)estimated by the value $\cos\varphi = 0.9$

Network analysis

- Determination of:
  - *voltages* at every node
  - *line currents* in every branch
  - *losses* in every branch
  - check of the *constraints* imposed to the system (e.g., losses and currents not higher than a given threshold value, voltages belonging to the admissible interval of values, etc…)
- Various *software* tools are used to calculate, at a given instant, all the electrical quantities of interest and to check the constraint satisfaction
- If the constraints are not respected, the *control variables* (HV-side supply voltage, transformation ratio variable under load of the HV/MV transformers, possible node capacitors) or the *structure* of the network are modified, opening some branches and closing other branches to restore the radial configuration for all the supply paths to the load nodes

Model of the components

- Generators:
  - *represented* in the HV supply nodes (with explicit model of the HV/MV transformer) or at the MV side (without explicit model of the HV/MV transformer), or modeling local generators
  - with *more HV/MV substations*, each network is analyzed separately
  - with a single radial network the generator (0) maintains the voltage constant in amplitude, and serves as phase angle reference
  - the corresponding voltage is $\underline{V}_0 = V_0 \, e^{j0}$
- Electrical lines and transformers:
  - electric lines are represented by the π equivalent circuit with lumped parameters (RX_L series parameters, and shunt parameters composed of the capacitive susceptance B_C)
  - transformers of the HV/MV substations and transformers associated to local generators are represented with the double bipole classical model, with series and shunt parameters
  - MV/LV substation transformers typically are not included in the model

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Steady-state load models

- Different models for *steady-state* studies are possible according to the type of load to represent
- Let's consider the subscript 0 to indicate *rated* or *reference conditions*
- The common *ZIP load representation* contains three types of load models:
  - a) assigned *impedance* (modulus $Z = Z_0$ and assigned power factor)
  - b) assigned *current* (amplitude $I = I_0$ and assigned *power factor*)
  - c) assigned *power* ($P = P_0$, $Q = Q_0$)
- A more general representation depending on user-defined *exponents* is

$$P = P_0 \, (V/V_0)^{\alpha} \qquad Q = Q_0 \, (V/V_0)^{\beta}$$

and varying the exponents α and β it is possible to obtain the previous models:

$\alpha = \beta = 2 \rightarrow$ back to the model a)
$\alpha = \beta = 1 \rightarrow$ back to the model b)
$\alpha = \beta = 0 \rightarrow$ back to the model c)

or hybrid models (e.g., $\alpha = \beta = 1.5$, cases with $\alpha \neq \beta$, or even negative exponents)

Steady-state load models

- The above indicated models can be combined into *polynomial* forms
- The load dependence on the *system frequency* can be represented explicitly, multiplying the load by a factor $(1 + \gamma (f - f_0))$, where f is the actual frequency, f_0 is the rated frequency and γ is the load sensitivity to the system frequency
- The *EPRI LOADSYN model* is a widely used model that summarizes the characteristics of the previously indicated formulations
- The *active power load* is divided into *two fractions*: the fraction P_{a1} depends on frequency with sensitivity *KPF1* and on voltage (exponent *KPV1*); the complementary fraction depends on voltage (exponent *KPV2*)
- The *reactive power load*, having initial reactive power Q' without compensation, is divided into two terms: one (with parameters $Q_{a1} = Q'/P_0$, *KQF1* and *KQV1*) refers to all load components, the other (with parameters *KQF2* and *KQV2*) approximates the effect of reactive "losses" and compensation in the subtransmission and distribution system

EPRI LOADSYN model

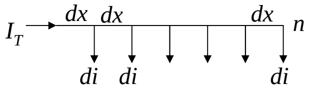
- Formulation of the *EPRI LOADSYN model*

$$P = P_0 \left[\underbrace{P_{a1} \left(\frac{V}{V_0} \right)^{KPV1} \left(1 + KPF1 \left(f - f_0 \right) \right)}_{\text{frequency-dependent load models}} + \underbrace{\left(1 - P_{a1} \right) \left(\frac{V}{V_0} \right)^{KPV2}}_{\text{frequency-independent load models}} \right]$$

$$Q = Q_0 \left[\underbrace{Q_{a1} \left(\frac{V}{V_0} \right)^{KQV1} \left(1 + KQF1 \left(f - f_0 \right) \right)}_{\text{reactive power of all load components}} + \underbrace{\left(\frac{Q_0}{P_0} - Q_{a1} \right) \left(\frac{V}{V_0} \right)^{KQV2} \left(1 + KQF2 \left(f - f_0 \right) \right)}_{\text{effect of reactive ``losses'' and compensation in the networks}} \right]$$

Uniformly distributed load

- Representation of a feeder of length ℓ and total current I_T with similar loads
- Hypotheses:
  - n loads, $dx = \ell/n$, $di = I_T/n$
  - impedance $Z = z\,\ell$
  - equal power factor for each current



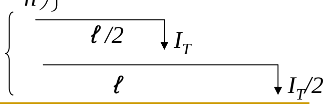
- Searching for an *equivalent lumped load* representation: at what distance has the equivalent load to be introduced?
  - a) criterion of maintaining the same *voltage drop* $\Delta V \approx \mathrm{Re}\{\bar{Z}\,\bar{I}\}$
    - first segment $\Delta V_1 = \mathrm{Re}\{z\,dx\,n\,di\}$
    - second segment $\Delta V_2 = \mathrm{Re}\{z\,dx\,(n-1)\,di\}$
    - total voltage drop $\Delta V_{TOT} = \mathrm{Re}\{z\,dx\,di\,[n+(n-1)+\ldots+1]\}$

    Since $1+2+3+\ldots+n = n(n+1)/2$

$$\Delta V_{TOT} = \mathrm{Re}\{z\,dx\,di\,n(n+1)/2\} = \mathrm{Re}\left\{\frac{1}{2} Z\,I_T \left(1+\frac{1}{n}\right)\right\}$$

$$\text{for } n \to \infty : \Delta V_{TOT} = \mathrm{Re}\left\{\frac{1}{2} Z\,I_T\right\}$$

alternatives



Uniformly distributed load

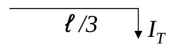
b) criterion of maintaining the same *total branch losses*

- first segment $\Delta P_1 = 3\,r\,dx\,|n\,di|^2$
- second segment $\Delta P_2 = 3\,r\,dx\,|(n-1)\,di|^2$
- total losses $\Delta P_{TOT} = 3\,r\,dx\,|di|^2\left[n^2 + (n-1)^2 + \ldots + 1^2\right]$

  Since $1^2 + 2^2 + 3^2 + \ldots + n^2 = n(n+1)(2n+1)/6$

$$\Delta P_{TOT} = 3\,r\,\frac{\ell}{n}\left|\frac{I_T}{n}\right|^2\left(\frac{n(n+1(2n+1))}{6}\right) = 3\,R\,|I_T|^2\left[\frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right]$$

$$\text{for } n \to \infty:\ \Delta P_{TOT} = 3\,\frac{R}{3}\,|I_T|^2$$

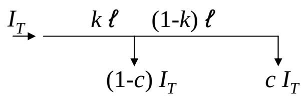


The equivalent representations for maintaining the same voltage drop or the same total branch losses are *different* !

It is *not possible* to use a lumped representation with a *single load*

Uniformly distributed load

- The exact model working for all cases has *two lumped loads*, such that



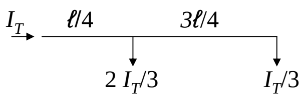
The coefficients *c* and *k* are obtained by considering

$$\Delta V_{TOT} = \mathrm{Re}\left\{\frac{1}{2}Z\,I_T\right\} = \mathrm{Re}\{k\,Z\,I_T + (1-k)Z\,c\,I_T\} \Rightarrow \frac{1}{2} = k + (1-k)c \Rightarrow k = \frac{1/2 - c}{1-c}$$

$$\Delta P_{TOT} = 3\left[\frac{R}{3}|I_T|^2\right] = 3\left[k\,R\,|I_T|^2 + (1-k)R\,|c\,I_T|^2\right] \Rightarrow \frac{1}{3} = k + (1-k)c^2 = k(1-c^2) + c^2$$

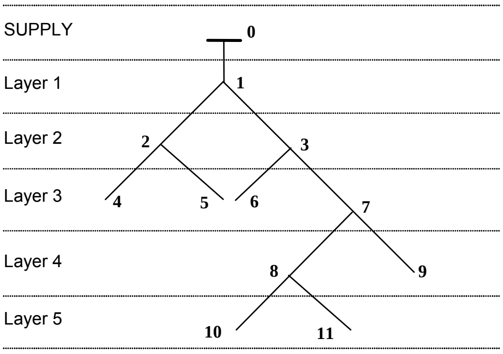
After substituting the expression of *k* into the last equation $\Rightarrow$ $c = \frac{1}{3}$

Thus, $k = \frac{1}{4}$ and the final solution for the circuit is



Distribution system structure

- The electricity distribution system structure can be considered as stratified into *layers* to simplify its numerical treatment
- The layer representation of the distribution system structure includes the node-to branch *incidence matrix* (L) and its *inverse* (Γ)
- Both matrices can be built by *visual inspection*



$$L=\begin{bmatrix} -1 & & & & & & & & & & \\ 1 & -1 & & & & & & & & & \\ 1 & & -1 & & & & & & & & \\ & 1 & & -1 & & & & & & & \\ & 1 & & & -1 & & & & & & \\ & & 1 & & & -1 & & & & & \\ & & 1 & & & & -1 & & & & \\ & & & & & & 1 & -1 & & & \\ & & & & & & 1 & & -1 & & \\ & & & & & & & 1 & & -1 & \\ & & & & & & & 1 & & & -1 \end{bmatrix}$$

$$\Gamma=\begin{bmatrix} -1 & & & & & & & & & & \\ -1 & -1 & & & & & & & & & \\ -1 & & -1 & & & & & & & & \\ -1 & -1 & & -1 & & & & & & & \\ -1 & -1 & & & -1 & & & & & & \\ -1 & & -1 & & & -1 & & & & & \\ -1 & & -1 & & & & -1 & & & & \\ -1 & & -1 & & & & -1 & -1 & & & \\ -1 & & -1 & & & & -1 & & -1 & & \\ -1 & & -1 & & & & -1 & -1 & & -1 & \\ -1 & & -1 & & & & -1 & -1 & & & -1 \end{bmatrix}$$

Distribution system representation

- Other relevant matrices and vectors are
  - the (diagonal) matrix $\mathbf{Z}_B$ containing the branch impedances
  - the vector $\mathbf{i}_S$ of the node currents, conventionally containing all the *output* currents from the system nodes

$$\mathbf{Z}_B=\begin{bmatrix} \underline{Z}_1 & & & & & & & & \\ & \underline{Z}_2 & & & & & 0 & & \\ & & \underline{Z}_3 & & & & & & \\ & & & \underline{Z}_4 & & & & & \\ & & & & \underline{Z}_5 & & & & \\ & & & & & \underline{Z}_6 & & & \\ & 0 & & & & & \underline{Z}_7 & & \\ & & & & & & & \dots & \\ & & & & & & & & \dots \end{bmatrix} \qquad \mathbf{i}_S=\begin{bmatrix} \underline{I}_{S1} \\ \underline{I}_{S2} \\ \underline{I}_{S3} \\ \underline{I}_{S4} \\ \underline{I}_{S5} \\ \underline{I}_{S6} \\ \underline{I}_{S7} \\ \dots \\ \dots \end{bmatrix}$$

Distribution system load flow

- single-phase *equivalent* load flow (for *balanced* three-phase systems)
- three-phase load flow (for *unbalanced* systems)
- probabilistic load flow (with *uncertain* data)
- harmonic load flow (with *distorted waveforms*)

Load flow for balanced systems

- Generally, the *currents* contained in the vector $\mathbf{i}_S$ depend on the value of the node *voltage*, e.g.:
  - for a load with specified *impedance* $\underline{Z}_{Ci}$ at node i (e.g., power factor correction capacitor, or representation of the shunt branch parameters), the current is $\underline{I}_{Si} = \underline{V}_i / \underline{Z}_{Ci}$
  - for a load with specified *power* $\underline{S}_{Ci} = P_{Ci} + j\, Q_{Ci}$ at node i, the current is $\underline{I}_{Si} = \underline{S}_{Ci}^* / \underline{V}_i^*$
- The initial values of the complex node voltages are fixed at each node $i = 1,\dots, n$, e.g., with values equal to the voltage at the *supply* node

$$\underline{V}_i^{(0)} = \underline{V}_0 = V_0\, e^{j0}$$

Backward/forward sweep method

- The load flow calculation is carried out by using an *iterative* procedure called Backward/Forward Sweep (BFS)
- The *k*-th iteration is composed of *two stages*
  - **Backward stage:** given the load data and complex voltages at the load terminals, compute the *complex branch currents*, starting from the load terminals and moving "backward" to the root
  - **Forward stage:** given the branch currents, compute the *complex voltages* at the load terminals, proceeding "forward" from the root to the load terminals
- The two stages are repeated iteratively, until the difference between the load voltages computed at the current iteration and at the previous iteration becomes lower than a specified *tolerance*, thus leading to convergence

Backward stage

- At the iteration *k*, the components of the node current vector $\mathbf{i}_S^{(k)}$ are
  - For the impedance-specified load at node *i,* with impedance $\underline{Z}_{Ci}$

$$\underline{I}_{Si}^{(k)} = \underline{V}_i^{(k\text{-}1)} / \underline{Z}_{Ci}$$

  - For the power-specified load at node *h,* with complex power $\underline{S}^*_{Ch}$

$$\underline{I}_{Sh}^{(k)} = \underline{S}^*_{Ch} / \underline{V}^*_h{}^{(k\text{-}1)}$$

- The *branch current vector* $\mathbf{i}_\mathbf{B}$ is then computed as

$$\mathbf{i}_B^{(k)} = \Gamma^\mathrm{T}\, \mathbf{i}_S^{(k)}$$

Forward stage

- At the iteration k, the node voltages are computed starting from the voltage V_0 at the *root node* by using the relationship

$$\mathbf{v}^{(k)} = V_0\, \mathbf{1} - \Gamma\, \mathbf{Z}_B\, \mathbf{i}_B^{(k)}$$

- where
  - **1** is a column vector with *all unity components*
  - the matrix Γ practically represents a "*filter*" applied to the matrix $\mathbf{Z}_B$ to consider, for each node, only the impedances located in the path from that node and the root
  - the vector $\Gamma\, \mathbf{Z}_B\, \mathbf{i}_B^{(k)}$ gives for each node the *voltage drop* occurring from the root node to the specified node

Stop criterion

- For each node, the following *difference* is considered between the voltage computed at the current iteration and the voltage at the previous iteration:

$$\max_i \left\{ \frac{\left| \underline{V}_i^{(k+1)} - \underline{V}_i^{(k)} \right|}{V_i^{(k)}} \right\} < \varepsilon \qquad \text{for } i = 1, \ldots, n$$

- The iterative process *terminates* when the maximum relative error (for the node in which the error is maximum) is lower than the specified threshold ε, otherwise the iterations continue

BFS as a Gauss method

- The *two stages* of the backward/forward sweep method can be merged to obtain the formulation

$$\mathbf{v}^{(k)} = V_0 \, \mathbf{1} - \Gamma \, \mathbf{Z}_B \, \Gamma^T \, \mathbf{i}_S^{(k)}$$

- Every node current is a *function* of the corresponding node voltage computed at the previous iteration

$$\mathbf{i}_S^{(k)} = g(\mathbf{v}^{(k-1)})$$

- The BFS method can then be seen as a *Gauss-type* numerical method, where

$$\mathbf{v}^{(k)} = f(\mathbf{v}^{(k-1)})$$