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REFRACTION OF TEM WAVES AT THE INTERFACE BETWEEN A CHARGED DIELECTRIC AND A LIQUID CONDUCTOR

BY

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Abstract. The results of an analysis of TEM waves impinging normally on the plane boundary between a gas containing free charges and a liquid conductive material are presented. The numerical tests show that only waves in the LF band (and lower) are influenced by the presence of free space charges in the gaseous dielectric.

Key words: surface resistivity; electrostatic charge and control.

1. Introduction

Electromagentic wave propagation in real dielectric or conducting motionless media is governed by Maxwell’s equations which lead to hyperbolic or parabolic partial differential equations (PDEs) in the time domain or to Helmholtz PDEs in the frequency domain. In the case of discontinuities in material properties, reflections and refractions occur and the characteristic parameters such as complex propagation constants, reflection and transmission coefficients, voltage standing wave ratio (in the medium where the wave originates) are functions of the electric and magnetic constants of the two

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materials (Miner, 1996; Petrescu, 2010). One interesting case is that when one or both media have a non-zero charge density or when surface charges are present on the boundary (Bektas *et al.*, 1986). An analysis of plane waves at solid–gas and fluid–solid interfaces in the presence of charges has been reported in literature (Bektas *et al.*, 1986; Poncelet *et al.*, 1994).

The present paper studies the case of TEM waves impinging normally on the plane boundary between a gas containing free charges and a liquid material whose conductive quality is frequency dependent (good conductor at small frequencies and weak conductor at large frequencies). This case presents importance in the study of waves propagating through air, which normally presents ions with a concentration that depends on weather conditions and the time of day, waves that penetrate through water (usually sea-water which is conductive). A numeric simulation is performed in order to establish the influence of air ion concentration and wave frequency on the reflection coefficient, wavelength, impedance and voltage standing wave ratio.

2. Problem Formulation

Since the parameters of both media intervene in the expressions of the reflection and refraction coefficients and in that of the voltage standing wave ratio, they are firstly determined for each of the two media separately.

2.1. Dielectric with Free Space Charges

The case of a dielectric containing free space charges is firstly considered. The volume charge density is $\rho_v = Nq$, where N is the number of charges in the unit volume (1 m^3) and q – the elementary charge (considering that only ions with this equivalent charge are present).

If an electromagnetic field is established in the dielectric, an electric force $\mathbf{F}_{\mathrm{el}} = q\mathbf{E}$ and a magnetic one $\mathbf{F}_{\mathrm{mg}} = q\mathbf{v} \times \mathbf{B}$ act on the elementary charge so that the total force is

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right). \tag{1}$$

However, in the case of planar harmonic electromagnetic waves $\underline{E} = \underline{\zeta}\,\underline{H} = \underline{\zeta}\,\underline{B}/\mu$, where $\underline{\zeta}$ is the wave impedance of the dielectric. In air, for example, $\zeta = 377\ \Omega$ and $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m, so that

$$\frac{\left|\mathbf{F}_{\mathrm{el}}\right|}{\left|\mathbf{F}_{\mathrm{mg}}\right|} = \frac{\zeta}{v\mu} = \frac{3.0001 \times 10^8}{v}.$$

It follows that only for speeds close to the speed of light, $c = 3 \times 10^8$ m/s, the modules of the two forces, $\mathbf{F}_{\mathrm{el}}$ and $\mathbf{F}_{\mathrm{mg}}$, are of the same order of magnitude, while for $v << c$, $F_{\mathrm{el}} >> F_{\mathrm{mg}}$. In the subsequent discussion only the electric force is considered.

Thus the forces acting on the charged particle of mass m (neglecting the gravitational force) satisfy the balance equation:

$$\mathbf{F}_{\mathrm{el}} = m\mathbf{a} = m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}. \tag{2}$$

The shift of these charges represents a convection current with the density $\mathbf{J}_d = \rho_v \mathbf{v}$. For high frequency fields, using the complex representation, Ampere's circuital law has the expression

$$\mathrm{rot}\ \underline{\mathbf{H}} = \rho_v \mathbf{v} + \mathrm{j}\omega\varepsilon\underline{\mathbf{E}}. \tag{3}$$

Relation (2), written in complex values, becomes:

$$q\underline{\mathbf{E}} = \mathrm{j}\omega m\mathbf{v}, \tag{4}$$

so that, substituting $\mathbf{v}$ in (3) it results

$$\mathrm{rot}\ \underline{\mathbf{H}} = -\mathrm{j}\frac{\rho_v q}{\omega m}\underline{\mathbf{E}} + \mathrm{j}\omega\varepsilon\underline{\mathbf{E}}\,. \tag{5}$$

In a charge free lossy dielectric Ampere's circuital law would be

$$\mathrm{rot}\ \underline{\mathbf{H}} = \mathrm{j}\omega\varepsilon_{\mathrm{eff}}\,\underline{\mathbf{E}}\,. \tag{6}$$

Comparing (5) and (6) leads to the conclusion that the presence of the electric charges modifies the permittivity, namely

$$\varepsilon_{\mathrm{eff}} = \varepsilon - \frac{\rho_v q}{\omega^2 m}. \tag{7}$$

Applying the rotor operator to relation (6) and taking into account that $\mathrm{div}\,\underline{\mathbf{H}} = 0$ a complex Helmholtz PDE is obtained

$$\Delta\underline{\mathbf{H}} + \omega^2\mu\varepsilon_{\mathrm{eff}}\,\underline{\mathbf{H}} = 0. \tag{8}$$

In the case of a uniform plane wave propagating in the Oz direction, the magnetic field $\underline{\mathbf{H}} = \mathbf{j}H(z)$ satisfies the ordinary differential equation:

$$\frac{\mathrm{d}^2\underline{H}}{\mathrm{d}z^2} + \omega^2\mu\varepsilon_{\mathrm{eff}}\,\underline{H} = 0, \tag{9}$$

having the solution

$$\underline{H}(z) = H_d \mathrm{e}^{-\underline{\gamma} z} + H_i \mathrm{e}^{\underline{\gamma} z}, \tag{10}$$

where $\underline{\gamma} = \mathrm{j}\omega\sqrt{\mu\varepsilon_{\text{eff}}} = \mathrm{j}\beta$ is the complex propagation constant and H_d , H_i represent the r.m.s. values of the direct and inverse magnetic field components, respectively. The phase constant, β, has the expression,

$$\beta = \omega\sqrt{\mu\varepsilon_{\text{eff}}} = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \frac{\rho_v q}{\omega^2 \varepsilon m}}. \tag{11}$$

In order to take place the propagation, β must be real, so that only waves having the angular frequency larger than a critical value

$$\omega > \omega_{\text{cr}} = \sqrt{\frac{\rho_v q}{\varepsilon m}}, \tag{12}$$

can propagate. The dielectric with free volume charge density presents no losses since $\alpha = \Re e\left\{\underline{\gamma}\right\} = 0$.

The electric field is given by the relation:

$$\underline{\mathbf{E}} = \frac{1}{\mathrm{j}\omega\varepsilon_{\text{eff}}} \operatorname{rot} \underline{\mathbf{H}} = -\frac{1}{\mathrm{j}\omega\varepsilon_{\text{eff}}} \cdot \frac{\partial \underline{H}}{\partial z}\mathbf{i} = \sqrt{\frac{\mu}{\varepsilon_{\text{eff}}}}\left(H_d \mathrm{e}^{-\underline{\gamma} z} - H_i \mathrm{e}^{\underline{\gamma} z}\right)\mathbf{i} = \underline{\mathbf{E}}_d + \underline{\mathbf{E}}_i . \tag{13}$$

The wave impedance of the dielectric with volume free charges is

$$\underline{\zeta} = \sqrt{\frac{\mu}{\varepsilon_{\text{eff}}}} = \sqrt{\frac{\mu}{\varepsilon - \dfrac{\rho_v q}{\omega^2 m}}} \in \mathbb{R}_+ . \tag{14}$$

It may be observed that $\zeta > \zeta_0$, where $\zeta_0 = \sqrt{\mu/\varepsilon}$ is the wave impedance in the absence of the space charge. It is also to be noticed that for a large enough space charge,

$$\rho_v > \frac{\omega^2 m\varepsilon}{q}, \tag{15}$$

the effective permittivity becomes negative. Indeed, while most dielectrics have only positive permittivities, plasmas exhibit negative permittivity values in certain frequency bands. Moreover, synthetic materials, known as *metamaterials*, can display negative values for both permittivity and permeability in a common range of frequencies (http://en.wikipedia.org/…).

2.2. Weak Conductor

The case of a weak conductor medium with the material constants ε, μ, $\sigma \neq 0$ is next considered. The electric field satisfies the Helmholtz PDE

$$\Delta\underline{\mathbf{E}} - \mathrm{j}\sigma\mu\left(\sigma + \mathrm{j}\omega\varepsilon\right)\underline{\mathbf{E}} = 0, \tag{16}$$

with the solution $\underline{\mathbf{E}}(z) = \left(\underline{E}_d \mathrm{e}^{-\underline{\gamma} z} + \underline{E}_i \mathrm{e}^{\underline{\gamma} z}\right)\mathbf{i}$, where

$$\underline{\gamma} = \alpha + \mathrm{j}\beta = \sqrt{\mathrm{j}\omega\mu\left(\sigma + \mathrm{j}\omega\varepsilon\right)} \tag{17}$$

is the complex propagation constant. In this case the attenuation and the phase constant are

$$\alpha = \frac{\omega\mu\sigma}{2\beta}, \quad \beta = \omega\sqrt{\mu\varepsilon}\sqrt{\frac{1 + \sqrt{1 + \sigma^2/\omega^2\varepsilon^2}}{2}}. \tag{18}$$

The wave impedance has the expression (Petrescu, 2010)

$$\underline{\zeta} = \frac{\omega\mu}{\beta - \mathrm{j}\alpha} = \frac{\underline{\gamma}}{\sigma + \mathrm{j}\omega\varepsilon} = \sqrt{\frac{\mathrm{j}\omega\mu}{\sigma + \mathrm{j}\omega\varepsilon}}, \tag{19}$$

being complex.

2.3. Charged Dielectric in Contact with a Weak Conductor

Finally let us consider the case of two media, a dielectric with free volume charges and a weak conductor, separated by a plane surface. A uniform TEM wave is normally incident on the plane boundary between the two regions (Fig.1). In this case a direct and a reflected (inverse) wave appear in the first medium and a transmitted (direct) wave in the second medium.

The reflection coefficient has the expression (Miner, 1996)

$$\underline{\Gamma} = \left.\frac{\underline{E}_i^{(1)}}{\underline{E}_d^{(1)}}\right|_{z=0} = \frac{\underline{\zeta}_2 - \underline{\zeta}_1}{\underline{\zeta}_2 + \underline{\zeta}_1}, \tag{20}$$

and the transmission coefficient is

$$\underline{\tau}=\left.\frac{\underline{E}_d^{(2)}}{\underline{E}_d^{(1)}}\right|_{z=0}=1+\underline{\Gamma}=\frac{2\underline{\zeta}_2}{\underline{\zeta}_1+\underline{\zeta}_2}, \tag{21}$$

where the indexes 1 and 2 refer to the two media.

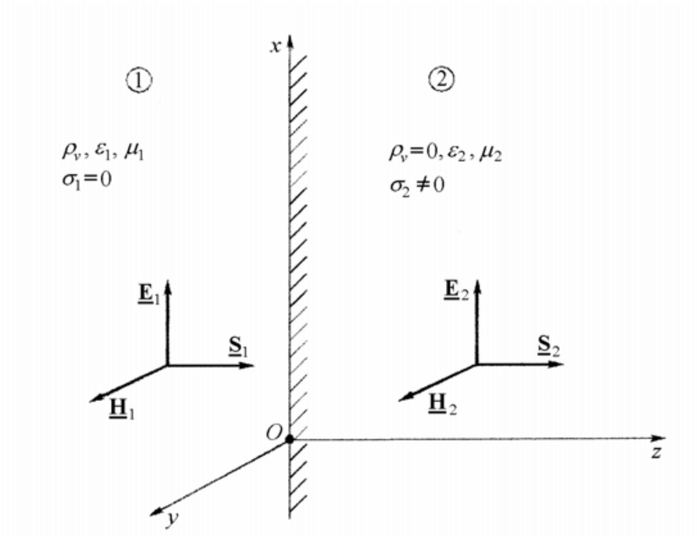

Fig. 1 – TEM wave with normal incidence.

The interference of the direct and inverse waves in the first medium produces standing waves with an amplitude that varies periodically along the *Oz*-axis. The distance between two neighbouring points with maximum and minimum amplitude is $\pi/2\beta_1$ and the voltage standing wave ratio (VSWR) is (Miner, 1996)

$$s=\frac{\max\{|\underline{E}_1|\}}{\min\{|\underline{E}_1|\}}=\frac{1+|\underline{\Gamma}|}{1-|\underline{\Gamma}|}, \tag{22}$$

because $\alpha_1 = 0$ (medium 1 is lossless).

3. Numerical Results and Discussions

The case of a uniform plane wave propagating through air and having a normal incidence on the plane surface of the sea water is considered. Air above the earth (either land or water) is usually, during fine weather, positively electrified, with an ion density that varies in the range

60…600 ions/cm^3 (Klusek, *et al.*, 2004). In this paper a medium concentration $N = 200$ ions/cm^3 was considered, corresponding to August mid-day recordings above the Baltic Sea (Klusek, *et al.*, 2004). The ions have a positive elementary charge, $q = = 1.602176487 \times 10^{-19}$ C (proton charge). The sea water was considered to be free of charge, having the conductivity $\sigma_2 = 4.8$ S/m (mean value) and the permitivity $\varepsilon_2 = 80.1\varepsilon_0$. The air permittivity was considered $\varepsilon_1 = 1.00058986\varepsilon_0$ and the proton mass $m = 1.672621637 \times 10^{-27}$ kg.

Several simulations were carried out to determine the wave propagation characteristic parameters (complex propagation constants, wave impedances, wavelengths, reflection coefficient and VSWR) for three frequencies ($f = 10^4$, 10^6 and 10^8 Hz) and in the presence or in the absence of air space charges. The obtained results are presented in Table 1.

Table 1
Wave Characteristic Parameters

f, [Hz]	10^4	10^4	10^6
ρ_v	0	$200 \times 10^6 q$	0
$\underline{\gamma}_1$	j 2.0950 × 10^{-4}	j 2.00084 × 10^{-4}	j 0.0209501
ζ_1, [Ω]	376.8799	394.6181	376,8799
$\underline{\gamma}_2$	0.4353 + j 0.4353	0.4353 + j 0.4353	4.3511 + j 4.3551
ζ_2, [Ω]	0.0906 + j0.0906	0.0906 + j0.0906	0.9073 + j 0.9064
$\underline{\Gamma}$	–0.99951 + j 0.00048	–0.99954 + j 0.00045	–0.99518 + j 0.00478
s	4,155.67	4,351.26	415.379
λ_1, [m]	29,991.156	31,402.714	299.911
λ_2, [m]	14.4336	14.4336	1.4427
f, [Hz]	10^6	10^8	10^8
ρ_v	$200 \times 10^6 q$	0	$200 \times 10^6 q$
$\underline{\gamma}_1$	j 0.209500	j 2.095012	j 2.095012
ζ_1, [Ω]	376.8876	376.8799	376.8799
$\underline{\gamma}_2$	4.3511 + j 4.3551	41.5621 + j 45.5935	41.5621 + j 45.5935
ζ_2, [Ω]	0.9073 + j 0.9064	9.4580 + j 8.6218	9.4580 + j 8.6218
$\underline{\Gamma}$	–0.99518 + j 0.00478	–0.95006 + j 0.0435	–0.95006 + j 0.0435
s	415.381	39.8682	39.8682
λ_1, [m]	299.912	2.99911	2.99911
λ_2, [m]	1.4427	0.1378	0.1378

Some observations can be made by analysing the data in Table 1 namely

a) All the analysed physical quantities are frequency dependent and are slightly influenced by the presence of space charges in air.

b) In non-ionized air the phase constant, β_1 , has a slightly larger value than in ionized air, an effect that disappears at high frequencies. As a result the wavelength, λ_1 , is larger in the presence of space charges, but only for frequencies up to several MHz.

c) The wave impedance in non-ionized air is smaller than in ionized air, the effect being noticeable only for frequencies below 10 MHz.

d) The complex propagation constant module in the conductive sea-water, γ_2 , increases approximatively with one order of magnitude for a frequency increase of two orders of magnitude.

e) The sea water wave impedance, ζ_2 , is complex and displays the same type of frequency dependence as γ_2 .

f) The presence of air space charges has a slight influence on the complex reflection coefficient, leading to an increase of its real part and a decrease of its imaginary part, but noticeable only at lower frequencies (below 1 MHz).

g) The voltage standing wave ration, *s*, increases in ionized air, an effect that is prominent at 10 kHz, slightly noticeable at 1 MHz and negligible at 100 MHz. At the same time VSMR decreases approximatively by one order of magnitude when the frequency increases by two orders of magnitude.

The previous observations lead to the conclusion that a normal state of air ionization influences the reflection and transmission coefficients of LF (low frequency) waves in the RF (radio frequency) range and that waves, with frequencies above 100 MHz, are unperturbed by the presence of space charges (in normal concentration). This fact is of considerable importance in communications that use the upper part of the RF spectrum (VHF and UHF) and microwave frequencies.

4. Conclusions

The study conducted in this paper allows for an exact analysis of the propagation of uniform plane waves with normal incidence impinging on the plane boundary between a dielectric with free space charges and a weak conductor with no space charges. The practical case, considered in the study, when the two media are: ionized air and sea water, respectively, leads to the conclusion that the reflection and transmission coefficients and the voltage standing wave ratio are affected by the presence of space charges in the air only for frequencies in the lower part of the spectrum (up to MF waves).

REFERENCES

Bektas S.I., Farish O., Hizal M., *Computation of the Electric Field at a Solid/Gas Interface in the Presence of Surface and Volume Charges*. Phys. Sci. Meas. a. Instr., IEE Proc. A, **133**, *9*, 577-586 (1986).

Klusek Z., Wiszniewski A., Jakacki J., *Relationships between Atmospheric Positive*

Electric Charge Densities and Gas Bubble Concentrations in the Baltic Sea. Oceanologia, **46**, *4*, 459-476 (2004).

Miner G.F., *Lines and Electromagnetic Fields for Engineers*. Oxford Univ. Press, Oxford, UK, 1996.

Petrescu C., *High Frequency Electromagnetic Fields – Solved Problems* (in Romanian). Politehnium Publ. House, Iaşi, 2010.

Poncelet O., Deschamps M., *Reflection and Refraction of an Inhomogeneous Plane Wave on Fluid/Anisotropic Solid Interface*. Proc. of the IEEE Ultrasonics Symp., Cannes, **2**, Nov. 1-4, 753-756 (1994).

Thomas J.R., Ishimaru A., *Wave Packet Incident on Negative Index Media*. IEEE Trans. on Anten. a. Propag., **53**, *5*, 1591-1599 (2005).

* * * http://en.wikipedia.org/wiki/Negative_index_metamaterials.

STUDIUL REFRACŢIEI UNDELOR TEM LA FRONTIERA DINTRE UN MEDIU DIELECTRIC ÎNCĂRCAT CU SARCINĂ ELECTRICĂ ŞI UN CONDUCTOR LICHID

(Rezumat)

Se efectuează o analiză a undelor TEM cu incidenţă normală la suprafaţa de separaţie dintre un gaz ionizat şi un lichid slab conductor. Testele numerice au arătat că numai undele cu frecvenţe cuprinse în banda LF (sau mai joase) sunt influenţate de prezenţa stării de ionizare a dielectricului.