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A COMPARATIVE ANALYSIS OF WIND SPEED DISTRIBUTION EVALUATION

BY

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Abstract. A comparative analysis of some methods for estimating Weibull parameters is performed. The Weibull distribution is a widely used distribution, especially for modeling the random variable of wind speed. The techniques require historical wind speed data, collected over a certain time interval, to establish the parameters of the wind speed distribution for a particular location.

Key words: wind speed data; parameter estimation; Weibull distribution.

1. Introduction

World-wide utilization of renewable energy in electric power systems is growing rapidly due to environmental concerns and a constant increase of energy prices due to reducing the fossil fuels amount used to conventional energy sources. Wind power is one of the fastest growing electric generation technologies in the whole world. Wind generation brings a great amount of benefits to power systems, such as: a cheaper energy comparing with the thermal generation, emission reduction; wind energy is available for large areas, development of a wind power farm can be implemented much easier and faster than building a thermal or hydro plant, etc. Meantime, wind generation brings a

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series of difficulties to the traditional power systems, as: uncontrollability of power generation, the wind generation depending on wind availability, irregularly fluctuating and intermittence of power generation, respectively a poor predictability of the wind generation.

The wind property of interest when the power generation is analysed, is the wind speed. This is because the wind power output obtained from the wind is directly proportional to the cube of the wind speed. One of the main characteristics of wind is that this is highly variable and its properties vary from one location to another.

2. The Wind Speed Probability Density Function

Wind speed probability density function plays an important role in electric power generation applications of wind turbine. A large number of studies have been published in scientific literature related to wind energy, which propose to use a variety of probability density functions (*e.g.* normal, lognormal, gamma, Rayleigh, Weibull, etc.) to describe wind speed distributions (Villanueva & Feijoo, 2010; Carta & Ramirez, 2009; Lun & Lam, 2000). The common conclusion of these studies is that the Weibull distribution with two parameters may be successfully utilized to describe the principle wind speed variation. For account the variability of wind speed, it is assumed to be characterized by a Weibull distribution with a scale parameter, a, [m/s], and a shape parameter, b, (dimensionless). The Weibull probability density and cumulative distribution functions are given, respectively, by

$$f_w(v) = \frac{b}{a}\left(\frac{v}{a}\right)^{b-1} \exp\left[-\left(\frac{v}{a}\right)^{b}\right], \quad F_w(v) = 1 - \exp\left[-\left(\frac{v}{a}\right)^{b}\right]. \tag{1}$$

The most important requirements for effective wind power planning and operation in power systems is an accurate estimation of wind speed distribution. Investigation of wind generation integration must be carefully performed in accordance with the wind speed probabilistic character.

3. Methods of Parameters Estimation

The wind speed distribution is determined when its parameters are established. The parameters estimate of the Weibull distribution can be found using different estimation methods. Some methods are graphical and others are analytical, each method having a criterion which yields estimates that are best in some situations. The most commonly analytical methods are: maximum likelihood estimator (MLE), method of moments (MOM) and least squares method (LSM) (Razali *et al.*, 2009; Bain & Engelhardt, 1992).

Because the estimate parameters play a major role in developing a model of electric power wind generator, it is important that different estimation methods to be compared to fit parameters of Weibull distribution from wind speed database. In what follows we try to find an answer to the question: what method gives the best Weibull parameters estimation? For that, will be analysed the performance of these methods with the same wind speed database.

3.1. The Maximum Likelihood Estimator

The MLE is an analytical method, widely applied in engineering and mathematics problems. For our case, for Weibull distribution of wind speed, in accordance with MLE theory, the likelihood function is built as the joint density of the *n* random variables be a function of the unknown two parameters

$$L(a,b)=\prod_{i=1}^{n} f(v_i)=\prod_{i=1}^{n}\frac{b}{a}\left(\frac{v_i}{a}\right)^{b-1}\exp\left[-\left(\frac{v_i}{a}\right)^{b}\right], \tag{2}$$

where a and b values can be achieved by using iterative methods or limits method. Last method of parameters evaluation involves taking the partial derivatives of the likelihood function with respect to the parameters, setting the resulting equations equal to zero

$$\begin{gathered}\frac{\partial \ln L}{\partial b}=\frac{n}{b}+\sum_{i=1}^{n}\ln v_i-\frac{1}{a}\sum_{i=1}^{n}v_i^b\ln v_i=0,\\ \frac{\partial \ln L}{\partial a}=-\frac{n}{a}-\frac{1}{a^2}\sum_{i=1}^{n}v_i^b=0.\end{gathered} \tag{3}$$

The values of a and b result as solutions of these equations.

3.2. The Method of Moments

The MOM is another analytical method to establish the distribution parameters. If it is known the set of wind data, the moments of unknown parameters that depend on the two-parameter Weibull distribution will be equalized with the empirical moments, as in the following equations:

$$\bar{v}=\sum_{i=1}^{n}\frac{v_i}{n},\quad s=\frac{1}{n}\sum_{i=1}^{n}\left(v_i-\bar{v}\right)^2. \tag{4}$$

The analytical expression of mean and the variance of Weibull distributions are given by the following expressions:

$$M(v) = a\Gamma\left(1+\frac{1}{b}\right) \text{ and } D^2(v) = a^2\left\{\Gamma\left(1+\frac{2}{b}\right)+\left[\Gamma\left(1+\frac{1}{b}\right)\right]^2\right\}, \quad (5)$$

where Γ(…) is the gamma function.

We can obtain the b parameter from the coefficient of variation (by dividing the variance with the square mean) and, after that, the α parameter can be established based on first eq. (5).

3.3. Least Squares Method

For the estimation of Weibull parameters, the LSM is extensively used in engineering problems. The method provides a linear relation between the two parameters having as start point the twice logarithms of Weibull cumulative distribution function namely

$$\ln\ln\left[1-\frac{1}{F_w(v)}\right] = b\ \ln(v) - b\ \ln(a)\,. \quad (6)$$

This relationship represents a straight line, whose eq. is $Y = aX + b$, where

$$Y = \ln\ln\left[1-\frac{1}{F_w(v)}\right],\ X = \ln v,\ a = b \text{ and } b = -b\ \ln a\ . \quad (7)$$

Using the simple linear regression, the a and b parameters result from coefficients of polynomial linear fitting.

4. Case Study

In order to compare the methods above described, a Matlab program has been developed to evaluate the Weibull parameters, based on previous methods and same wind speed database. To evaluate the performance of these methods, the mean squared errors (MSE) has been used to evaluate the accuracy of estimated probability density function to real distribution. The MSE is a method to evaluate the difference between values provided by an estimated probability density function and the true values of the database distribution (Lange, 2005).

The wind data, used for analyses, was collected from the North-East area of Romania, over one year period, for the year 2008. The data collection was made at one hour interval, the hourly average values being recorded. Fig. 1 presents the wind speed values collected at anemometer height (10 m above the ground).

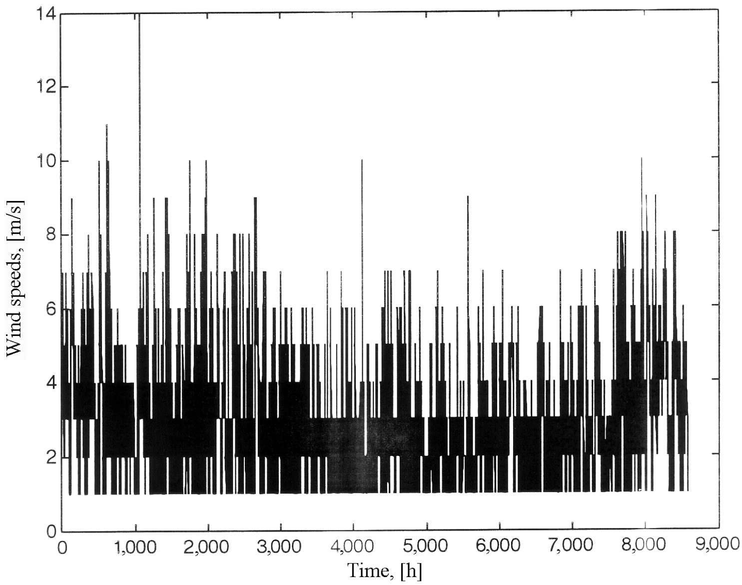

Fig. 1 – The hourly wind speed database.

Based on these measurements, the wind speed distribution and it parameters have been estimated, the Weibull distribution being a good approximation of the real database of wind speed frequency. The Table 1 shows the Weibull parameters for the analysed database, scale and shape parameters being determined using above methods.

Table 1
Weibull Parameters for Whole Year

Parameters/error	Method		
	MLE	MOM	LSM
Scale parameter, *a*	3.2260	3.2136	3.3143
Shape parameter, *b*	1.8661	1.8314	1.7898
MSE	0.1836	0.1790	0.2197

As can been seen, the parameters of Weibull distribution are very close, the scale parameter lies between 3.2136 and 3.3143 m/s, and the shape parameter between 1.7898 and 1.8661. Likewise, the last line of Table 1 shows the values of MSE for each method. It is found that MOM is superior in accuracy and has a smaller error compared with the MLE and LSM methods. The order of the methods based on their accuracy is as follows: MOM, MLE and LSM.

In order to compare the results of these estimation methods using different sample sizes of database, the above methods have been applied for the wind speed values from the months from each of the four seasons. The average seasonal Weibull parameters are presented in Table 2.

Table 2
Weibull Parameters for Seasons

Season	Parameters/error	Method		
		MLE	MOM	LSM
Spring	Scale parameter, *a*	3.2726	3.2590	3.2676
	Shape parameter, *b*	1.7869	1.7529	1.8033
	MSE	0.1706	**0.1651**	0.1709
Summer	Scale parameter, *a*	2.6723	2.6632	2.7337
	Shape parameter, *b*	1.9178	1.8854	1.8307
	MSE	0.2230	**0.2203**	0.2531
Autumn	Scale parameter, *a*	3.0376	3.0268	3.1265
	Shape parameter, *b*	1.9761	1.9434	2.0202
	MSE	0.2144	**0.2098**	0.2602
Winter	Scale parameter, *a*	4.0012	3.9968	4.1444
	Shape parameter, *b*	2.1214	2.1117	2.0446
	MSE	0.1486	**0.1477**	0.2092

As can been observed, the scale and shape parameters, from the whole database and from seasonal values, have the best estimation in case of MOM. Thus, we can say that MOM is the best method used to estimate the parameters for the two-parameter Weibull distributions taking into consideration the MSE as a measurement for comparison.

5. Conclusions

In practice, it is very important to describe the variation of wind speeds for optimal design of the wind generation systems. The wind variation for a typical site is usually described using the Weibull distribution. Therefore it is very important to know the best method for parameters evaluation, with minimal errors.

This study has been performed to compare the results of three methods of parameters estimation, for the same database. It has been deduced, from computational results, that method which gives the lowest values of MSE, is the MOM, in both cases, for whole year database and for seasonal values. However, from accuracy viewpoint, the LSM and the MLE of fitting Weibull function were also good methods because these ones give close values of parameters as MOM and, more over, Matlab package contains functions and tools that estimate the parameters and confidence intervals for Weibull data.

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REFERENCES

Bain L., Engelhardt M., *Introduction to Probability and Mathematical Statistics.* Duxbury Press, California, USA, 1992.

Carta J.A., Ramırez P., *A Review of Wind Speed Probability Distributions Used in Wind Energy Analysis. Case Studies in the Canary Islands.* Renew. a. Sustain. Energy Rev., **13**, *5*, 933-955 (2009).

Lange P.M., *On the Uncertainty of Wind Power Predictions – Analysis of the Forecast Accuracy and Statistical Distributions of Errors.* J. of Solar Energy Engng., **127**, 177-184 (2005).

Lun I.Y.F., Lam J.C., *A Study of Weibull Parameters Using Long-Term Wind Observations*. Renew. Energy J., **20**, *2*, 145-153 (2000).

Razali A.M., Salih A.A., Mahdi A.A., *Estimation Accuracy of Weibull Distribution Parameters*. J. of Appl. Sci. Res., **5**, *7*, 790-795 (2009).

Villanueva D., Feijoo A., *Wind Power Distributions: A Review of their Applications*. Renew. a. Sustain. Energy Rev., **14**, 1490–1495 (2010).

ANALIZA COMPARATIVĂ A METODELOR DE ESTIMARE A DISTRIBUŢIILOR PROBABILISTICE ALE VITEZEI VÂNTULUI

(Rezumat)

Estimarea distribuţiilor constituie tehnica prin care se determină valorile parametrilor distribuţiilor statistice, plecând de la un set de date obţinut pe cale experimentală, din statistici sau simulări. Sunt analizate trei metode de estimare a parametrilor funcţiilor de distribuţie (MLE, MOM şi LSM), particularizate pentru cazul distribuţiei Weibull, în vederea stabilirii metodei care furnizează cel mai ridicat grad de încredere asociat parametrilor distribuţiei. Cele trei metode au fost ierarhizate, din punct de vedere al preciziei, cu ajutorul metodei celor mai mici pătrate.

Metodele au fost evaluate pe aceeaşi bază de date, reprezentând valorile vitezei vântului, înregistrate în zona de nord-est a României, pentru anul 2008. Parametrii distribuţiei Weibull asociate bazei de date au fost evaluaţi cu cele trei metode, stabilind că metoda MOM conduce la cele mai bune rezultate.