BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi" din Iaşi
Tomul LVII (LXI), Fasc. 4, 2011
Secţia
ELECTROTEHNICĂ. ENERGETICĂ. ELECTRONICĂ

THE ACTIVE ENERGY TRANSMISSION EFFICIENCY THROUGH LINEAR, NON-AUTONOMOUS AND PASSIVE TWO-PORTS SUPPLYING NON-LINEAR INERTIAL AND PASSIVE RECEIVERS, IN HARMONIC STEADY-STATE (II)

BY

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Received, January 26, 2011
Accepted for publication: April 26, 2011

Abstract. Using the results established in a previous paper (Rosman, 2009), the problem of active energy transmission efficiency through linear, non-autonomous and passive two-ports is studied, in harmonic steady-state, when the receiver is constituted from the serial connexion of a resistor, a coil and a condenser, all three non-linear inertial and passive.

Key words: active energy transmission; efficiency; linear, non-autonomous and passive two-ports; non-linear inertial and passive receiver.

1. Introduction

In a previous paper (Rosman, 2009) the problem of the active energy transmission efficiency through a linear, non-autonomous and passive two-port (LNPT) in harmonic steady-state was studied, when the receiver is non-linear,

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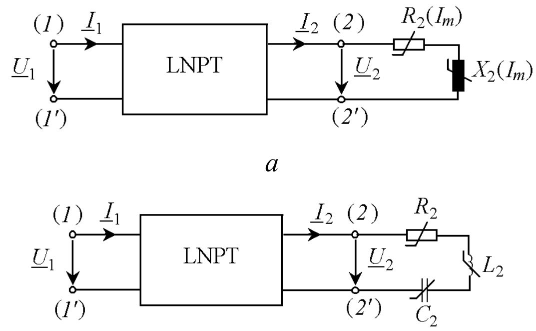
inertial and passive (NIPR). Namely, an LNPT is considered (Fig. 1 *a*), having the eqs.

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \end{bmatrix} = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{bmatrix} \begin{bmatrix} \underline{U}_2 \\ \underline{I}_2 \end{bmatrix}, \tag{1}$$

where $[\underline{A}_{ij}]$, $(i, j = 1, 2)$, is the fundamental parameters matrix supplying an NIPR, having the complex impedance

$$\underline{Z}_2(I_m) = R_2(I_m) + \mathrm{j}X_2(I_m), \tag{2}$$

I_m representing the amplitude of an arbitrary harmonic current.



a

b

Fig. 1

It was taked in account the fact that if a non-linear inertial element is supplied with a harmonic voltage the current which flows through this element is harmonic too (Philippow, 1963), while the current–voltage characteristic of such an element is non-linear in RMS values but linear in instantaneous values. So, if a harmonic voltage is applied at the input gate of an LNPT (Fig. 1 *a*), the established steady-state is a harmonic one; consequently the steady-state of such an LNPT may be performed, in this case, using the symbolic method of complex signals and parameters.

The main results obtained by the author (2009) may be summarized as follows:

1° The $X_2(R_2)$ function which corresponds to different active energy transmission states with extreme values of efficiency, for different values of reference current's amplitude, I_m, is

$$\begin{aligned} X_2(I_m) = \frac{1}{2\Re e(\underline{A}_{11}\underline{A}_{21}^*)} \Bigg(\Big\{ -4\big[\Re e(\underline{A}_{11}\underline{A}_{21}^*)\big]^2 R_2^2(I_m) + CR_2(I_m) + \\ + \big[\Re e(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}\underline{A}_{21}^*)\big]^2 + 1 \Big\}^{1/2} + \Im m(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}\underline{A}_{21}^*) \Bigg). \end{aligned} \tag{3}$$

2° The NIPR's equivalent resistance, R_2 , has values situated in the range

$$\begin{cases} R_{2\min} = \dfrac{C-\sqrt{C^2-16\left[\Re e(\underline{A}_{11}\underline{A}_{12}^{*})\right]^2\left\{1-\left[\Re e(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*})\right]^2\right\}}}{8\left[\Re e(\underline{A}_{12}\underline{A}_{22}^{*})\right]^2}, \\ R_{2\min} = \dfrac{C+\sqrt{C^2-16\left[\Re e(\underline{A}_{11}\underline{A}_{12}^{*})\right]^2\left\{1-\left[\Re e(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*})\right]^2\right\}}}{8\left[\Re e(\underline{A}_{12}\underline{A}_{22}^{*})\right]^2}, \end{cases} \tag{4}$$

where C is an integration constant.

3° The reactances which correspond to $R_{2\min}$, $R_{2\max}$ resistances, are equal, having the common value

$$X_2^{'} = X_2^{''} = X_2 = \frac{\Im m(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*})}{2\Re e(\underline{A}_{11}\underline{A}_{21}^{*})}. \tag{5}$$

4° The maximum efficiency, as function of $R_2(I_m)$ resistance, is given by relation

$$\eta_{\max}(I_m) = \frac{\Re e(\underline{A}_{11}\underline{A}_{21}^{*})R_2(I_m)}{\left[C/4+\Re e(\underline{A}_{11}\underline{A}_{21}^{*})\Re e(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*})R_2(I_m)\right]+2\Re e(\underline{A}_{11}\underline{A}_{21}^{*})\Re e(\underline{A}_{12}\underline{A}_{22}^{*})}, \tag{6}$$

graphically represented in Fig. 2.

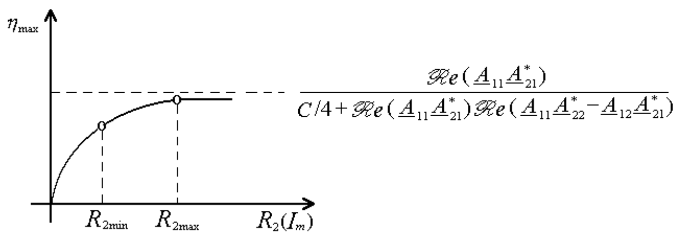


Fig. 2

5° The integration constant, C, satisfies the inequality

$$C \geq 4\Re e(\underline{A}_{11}\underline{A}_{21}^{*})\sqrt{1-\left[\Re e(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*})\right]^2}. \tag{7}$$

2. The Studied Particular Case

In what follows the particular case represented in Fig. 1 *b* is studied, the LNPT's receiver being a serial connexion of a resistor, a coil and a condenser, all three non-linear inertial. The parameters of such elements were established by Savin & Rosman (1993), namely

$$\begin{cases} R(I_m) = a_1 + \frac{3}{4}a_3 I_m^2, \ (a_1 > 0,\ a_3 \lesseqgtr 0), \\ X_L(I_m) = b_1\omega - \frac{3}{4}b_3\omega I_m^2, \ (b_1 > 0,\ b_3 \lesseqgtr 0), \\ X_C(I_m) = \frac{c_1}{\omega} + \frac{3}{4}\cdot\frac{c_3}{\omega^3} I_m^2, \ (c_1 > 0,\ c_3 \lesseqgtr 0), \end{cases} \tag{8}$$

In the case represented in Fig. 1 *b* the NIPR's parameters are therefore

$$\begin{cases} R_2(I_m) = a_1 + \frac{3}{4}a_3 I_{2m}^2, \\ X_L(I_{2m}) = X_L(I_{2m}) - X_C(I_{2m}) = b_1\omega - \frac{c_1}{\omega} - \frac{3}{4}\left(b_3\omega + \frac{c_3}{\omega^3}\right)I_{2m}^2. \end{cases} \tag{9}$$

Substituting expressions (9) in relation (3) it results the algebraic biquadratic eq.

$$\alpha I_{2m}^4 + \beta I_{2m}^2 + \gamma = 0, \tag{10}$$

where

$$\begin{cases} \alpha = \frac{9}{4}\left[\Re e(\underline{A}_{11}\underline{A}_{21}^*)\right]^2\left[a_3^2 + \left(b_3\omega + \frac{c_3}{\omega^3}\right)^2\right] > 0, \\ \beta = 6\left[\Re e(\underline{A}_{11}\underline{A}_{21}^*)\right]^2\left[a_1 a_3 - \left(b_1\omega - \frac{c_1}{\omega}\right)\left(b_3\omega + \frac{c_3}{\omega^3}\right) + \Im m(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}\underline{A}_{21}^*)\right] + \\ \quad + \frac{3}{4}Ca_3, \\ \gamma = 4\left[\Re e(\underline{A}_{11}\underline{A}_{21}^*)\right]^2\left[a_1^2 + \left(b_1\omega - \frac{c_1}{\omega}\right)^2 + 4\Re e(\underline{A}_{11}\underline{A}_{21}^*)\Re e(\underline{A}_{12}\underline{A}_{22}^*)\right] > 0. \end{cases} \tag{11}$$

The above expression of γ was established taken into account the identity (Rosman *et al*., 1965)

$$\begin{aligned}&\left[\Re e(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*})\right]^{2}+\left[\Im m(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*})\right]^{2}-\\&-4\Re e(\underline{A}_{11}\underline{A}_{21}^{*})\Re e(\underline{A}_{12}\underline{A}_{22}^{*})=1,\end{aligned} \tag{12}$$

satisfied by the fundamental parameters, $\underline{A}_{ij}$, (i, j = 1, 2), of any LNPT as well as (Şora, 1964)

$$\Re e(\underline{A}_{11}\underline{A}_{21}^{*})=\frac{P_{10}}{U_2^2}>0,\ \Re e(\underline{A}_{12}\underline{A}_{22}^{*})=\frac{P_{1\mathrm{sc}}}{I_2^2}>0. \tag{13}$$

Analysing the algebraic eq. (10) it may be observed that this one has at the most two real and positive roots if

$$\beta^2-4\alpha\gamma>0 \text{ and } -\beta\pm\sqrt{\beta^2-4\alpha\gamma}>0, \tag{14}$$

which implies

$$\beta<0. \tag{15}$$

It results that in case of an NIPR such that represented in Fig. 1 *b*, characterized by parameters a_i , b_i , c_i , (i = 1, 3), it exists at most two different harmonic steady-states for which the active power's transfer efficiency has extreme values.

It is possible to determine the current amplitudes which flow through the NIPR when his equivalent resistance has the values $R_{2\min}$, $R_{2\max}$. With this goal the expressions (4) of $R_{2\min}$ and $R_{2\max}$ are substituted in relation (9_1). Performing the calculus it results

$$\begin{aligned}&I_{2m}(R_{2\min})=\frac{1}{\sqrt{6a_3}\Re e(\underline{A}_{12}\underline{A}_{21}^{*})}\times\\&\times\sqrt{C-8a_1\left[\Re e(\underline{A}_{12}\underline{A}_{21}^{*})\right]^2-\sqrt{C^2-16\left[\Re e(\underline{A}_{12}\underline{A}_{21}^{*})\right]^2\left\{1-\left[\Re e(\underline{A}_{12}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*})\right]^2\right\}}},\\&I_{2m}(R_{2\max})=\frac{1}{\sqrt{6a_3}\Re e(\underline{A}_{12}\underline{A}_{21}^{*})}\times\\&\times\sqrt{C-8a_1\left[\Re e(\underline{A}_{12}\underline{A}_{21}^{*})\right]^2+\sqrt{C^2-16\left[\Re e(\underline{A}_{12}\underline{A}_{21}^{*})\right]^2\left\{1-\left[\Re e(\underline{A}_{12}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*})\right]^2\right\}}}.\end{aligned} \tag{16}$$

In a similar manner can be determined the current amplitude which flows through the NIPR when his equivalent reactance is $X_2^{'}=X_2^{''}=X_2$ (s.

relation (5)).

Along the same line it is possible to determine the active power's transfer maximum efficiency through an LNPT as function of current amplitude, I_{2m} , which flows through the NIPR. With this end in view in expression (6) of the maximum efficiency the resistance $R_2(I_m)$ is substituted with relation (9_1) resulting

$$\eta_{\max}(I_{2m}) = \frac{\delta + \varepsilon I_{2m}^2}{\varphi + \psi I_{2m}^2}, \tag{17}$$

where

$$\begin{cases} \delta = a_1 \Re e(\underline{A}_{11}\underline{A}_{21}^*) > 0, \ \varepsilon = \frac{3}{4} a_3 \Re e(\underline{A}_{11}\underline{A}_{21}^*), \\ \varphi = \left[\frac{C}{4} + \Re e(\underline{A}_{11}\underline{A}_{21}^*)\Re e(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}\underline{A}_{21}^*)\right] a_1 + 2\Re e(\underline{A}_{11}\underline{A}_{21}^*)\Re e(\underline{A}_{12}\underline{A}_{22}^*), \\ \psi = \frac{3}{4}\left[\frac{C}{4} + \Re e(\underline{A}_{11}\underline{A}_{21}^*)\Re e(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}\underline{A}_{21}^*)\right] a_3. \end{cases} \tag{18}$$

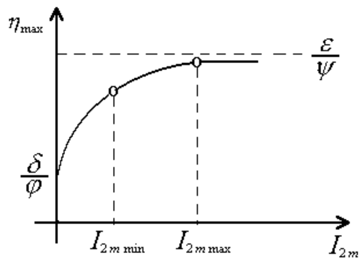


Fig. 3

The curve $\eta_{\max}(I_{2m})$ is represented in Fig. 3. Having in view that $\eta_{\max} > 0$ for any value of I_{2m} , hence for $I_{2m} = 0$ too, and that $\delta > 0$ it results that $\varphi > 0$. Inequality

$$\frac{\delta}{\varphi} < \frac{\varepsilon}{\psi} \tag{19}$$

represents a consequence of relations (13).

It is necessary to underline that only the curve's arc from Fig. 3 which corresponds to values of current amplitude I_{2m} situated in the range [$I_{2m\min}$, $I_{2m\max}$], has, as was proved in a previous paper (Rosman, 2009), a physical meaning.

3. Conclusions

Using the results obtained in a previous paper (Rosman, 2009) the problem of the active power's transfer efficiency, in harmonic steady-state through a linear, non-autonomous and passive two-port, is studied, when the receiver is constituted by the serial connexion of a resistor, a coil and a condenser, all three non-linear inertial. The main conclusions are the following:

1° It is possible to realize at most two different such regimes.

2° The maximum efficiency of active power's transfer is a function of the amplitude I_{2m}, represented in Fig. 3.

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RANDAMENTUL TRANSMISIEI ENERGIEI ACTIVE PRIN CUADRIPOLI LINIARI NEAUTONOMI ŞI PASIVI, ALIMENTÂND, ÎN REGIM PERMANENT ARMONIC, RECEPTOARE NELINIARE INERŢIALE (II)

(Rezumat)

Utilizând rezultatele stabilite într-o lucrare precedentă (Rosman, 2009) se studiază problema randamentului transmisiei energiei active printr-un cuadripol liniar, neautonom şi pasiv, alimentând, în regim permanent armonic, un receptor constituit din gruparea în serie a unui rezistor, unei bobine şi unui condensator, toate trei neliniare inerţiale.