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ASYMMETRIC TURBO CODED MODULATION IN MIMO SYSTEMS USING DOUBLY ITERATIVE DECODING ALGORITHM

BY

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Abstract. The performances of turbo coded modulation using asymmetric turbo codes with transmission and reception antenna diversity are analysed. We have considered that the component convolutional codes have the memory 2 and 3 (*i.e.* 4 and 8 states), and their generator feedback polynomials are both primitive and nonprimitive. To study both cases, simulations were performed to obtain the bit error rate (BER) and the frame error rate (FER). From simulation results it can be seen that those codes with primitive feedback polynomials lead to better performances for FER, whereas those with non-primitive ones lead to slightly improvements of BER in low SNR range.

Key words: space-time turbo codes; doubly iterative decoder; BER/FER performances.

1. Introduction

The spectral efficiency of wireless systems can be increased by using multiple antennas transmission techniques (Foschini Jr. & Gans, 1998; Telatar, 1995). The complexity of the optimal decoder is also exponentially increased

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with the modulation size and the antennas number, and it is important to design receiver interfaces with near-optimal performance with a moderate complexity.

Turbo codes with iterative decoding have been proposed by Berrou, Glavieux and Thitimajshima in 1993. In 2001 Stefanov and Duman introduced turbo-coded modulation with transmission and reception antenna diversity systems over block fading channels. The receiver complexity is increased by the large number of antennas, and to ensure a good compromise between complexity and performance a spatial interference canceling scheme was used (Biglieri *et al*., 2003). In 2005 Biglieri has presented a block scheme for a doubly iterative receiver with minimum mean square error (MMSE) algorithm.

In simulations, the component turbo decoder uses a maximum-logarithmic *a posteriori* probability (max-log-APP) algorithm and a quadratic permutation polynomial interleaver with the best Ω' metric (Ω'-QPP interleaver) (Takeshita, 2007).

The spreading factor, D, is defined by

$$D = \min_{\substack{i \neq j \\ i,j \in \mathbb{Z}_L}} \left\{ \delta_L \left(p_i, p_j \right) \right\}, \tag{1}$$

where $\delta_L\left(p_i, p_j\right)$ is the Lee metric between the points $p_i = \left(i, \pi(i)\right)$ and $p_j = \left(j, \pi(j)\right)$.

The metric Ω' is defined as being

$$\Omega' = \zeta' \ln D, \tag{2}$$

where ζ' is the refined nonlinearity degree (Takeshita, 2007).

The metric Ω' accomplishes a compromise between the multiplicity of code words, through the refined nonlinearity degree and the free distance of the turbo code, controlled by the spreading factor.

Takeshita (2007) has established the quadratic permutation polynomials (QPP) interleavers with the maximum D parameter (or spreading factor) and the maximum (the best) Ω' metric for some representative lengths. We choose a Ω'-QPP because this leads to an improved performance for large lengths. The free term is found by maximizing of the corner merit because only the first trellis is terminated, and the margin effects of trellis termination are avoided.

In Section **2** the models for the transmitter, channel and receiver are presented. Section **3** presents asymmetric turbo codes for antenna diversity systems. Simulation results are given in Section **4** and Section **5** concludes the paper.

2. System Model

A mobile communication system with N_t transmission antennas and N_r reception antennas is considered. The information bits (denoted by the vector $\boldsymbol{b}$)

are turbo-coded with coding rate R_c , and block size of N_t N, where N is the number of successive transmissions from the transmission antennas corresponding to a codeword. The interleaved bits which give the encoded vector, ***c***, are serial to parallel converted and mapped into a signal constellation. The signal at the modulator output, $x_{i,t}$, is transmitted by antenna i, ($1 \le i \le N_t$), at each time instant. It is chosen from a bidimensional constellation of size 2^M .

In this paper the QPSK (quaternary phase-shift keying) modulation is used. All signals are simultaneously transmitted from a different transmission antenna and have the same transmission period, T. The modulated signals sent on the channel are grouped in the space-time codeword matrix $\mathbf{X}=(x_1, x_2,\dots,x_N)$.

The received signal is a noisy superposition of the transmitted signals, corrupted by Rayleigh fading. $\alpha_{i,j}$ is the path gain from the transmission antenna, i, ($1 \le i \le N_t$), to the reception antenna, j, ($1 \le j \le N_r$). The fading is assumed to be block Rayleigh fading and the path gains can be modeled by complex independent Gaussian random variables with zero mean and variance 0.5 for each dimension.

The path gains are constant for L symbols corresponding to ηL information bits and independent from one L-symbols block to another. One codeword of the turbo-code contains $F = N_t N/(R_c ML)$ distinct blocks with constant fading.

The signal $y_{t,j}$, received by antenna j at time t, is

$$y_{t,j} = \sum_{i=1}^{N_t} \alpha_{i,j} x_{t,i} + z_{t,j} . \tag{3}$$

The noise samples, $z_{t,j}$, are modeled as independent realizations of a complex Gaussian random variable with variance $N_0/2$ for each dimension.

Equivalently, the received signal can be written as

$$\mathbf{y}_t = \mathbf{H}\mathbf{x}_t + \mathbf{z}_t, \tag{4}$$

with

$$\mathbf{y}_t = \begin{bmatrix} y_{t,1}, & y_{t,2}, & \dots & y_{t,N_r} \end{bmatrix}^T \qquad \qquad \mathbf{H} = \begin{bmatrix} \alpha_{1,1} & \alpha_{2,1} & \dots & \alpha_{N_t,1} \\ \alpha_{1,1} & \alpha_{2,2} & \dots & \alpha_{N_t,2} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{1,N_r} & \alpha_{2,N_r} & \dots & \alpha_{N_t,N_r} \end{bmatrix} . \tag{5}$$

$$\mathbf{x}_t = \begin{bmatrix} x_{t,1}, & x_{t,2}, & \dots & x_{t,N_t} \end{bmatrix}^T \quad \text{and}$$

$$\mathbf{z}_t = \begin{bmatrix} z_{t,1}, & z_{t,2}, & \dots & z_{t,N_r} \end{bmatrix}^T$$

Considering that there are F distinct blocks with constant fading, we can write the $N_r \times N$ matrix corresponding to the received signal as $\mathbf{Y}=(\mathbf{Y}_1, \dots, \mathbf{Y}_F)$. Similarily, $\mathbf{X}=(\mathbf{X}_1, \dots, \mathbf{X}_F)$ and $\mathbf{Z}=(\mathbf{Z}_1, \dots, \mathbf{Z}_F)$. For $\lambda = 1,\dots,F$, we have

$$\mathbf{Y}_\lambda = \mathbf{H}_\lambda \mathbf{X}_\lambda + \mathbf{Z}_\lambda. \tag{6}$$

Fig. 1 presents the transmitter block scheme, performing a coded modulation with bit interleaving and antenna diversity (Biglieri *et al*., 2005) and Fig. 2 shows the receiver block scheme (Biglieri *et al*., 2005) which uses a MMSE iterative algorithm.

The used turbo decoding algorithm is the max-log-APP and the turbo decoder uses a Ω'-QPP instead of random interleaver. The receiver uses a linear MMSE interface, consisting in a linear filter modeled by matrix **A** which minimizes the mean square error $E\|\mathbf{AY}-\mathbf{X}\|^2$, where $\|\cdot\cdot\|$ denotes the Frobenius norm,

$$\mathbf{A} = \left(\mathbf{H}^*\mathbf{H} + \frac{N_0}{E_s}\mathbf{I}_{N_t}\right)^{-1}\mathbf{H}^*. \tag{7}$$

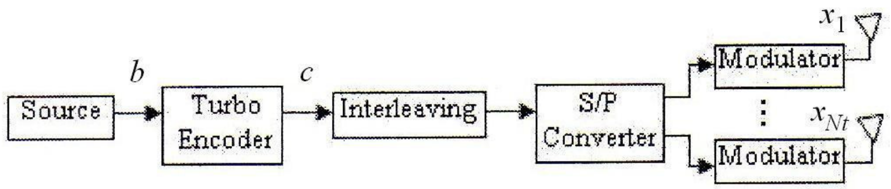

Fig. 1 – Transmitter block scheme.

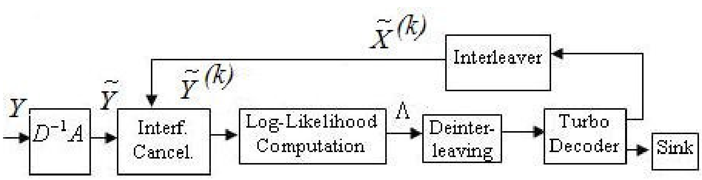

Fig. 2 – Receiver block scheme.

The normalized filtered signal is

$$\tilde{\mathbf{Y}} = \mathbf{D}^{-1}\mathbf{AY} = \mathbf{X} + \mathbf{LX} + \mathbf{GZ}, \tag{8}$$

where $\mathbf{D} = \mathrm{diag}(\mathbf{AH})$, $\mathbf{L} = \mathbf{D}^{-1}\mathbf{AH} - \mathbf{I}_{N_t}$ and $\mathbf{G} = \mathbf{D}^{-1}\mathbf{A}$.

For QPSK modulation, the soft estimates, $\tilde{x}_{i,t}$, provided back to the interference canceling block, are obtained considering the assumption above concerning the energy of a transmitted symbol. The estimates can be expressed as

$$\tilde{x}_{i,t} = \frac{1}{\sqrt{2N_t}}\left\{ \left[2P\left(c_{2i-1,t}=1\right)-1\right] + \mathrm{j}\left[2P\left(c_{2i,t}=1\right)-1\right] \right\} = \frac{1}{\sqrt{2N_t}}\left\{\tanh\left[\frac{L\left(c_{2i-1,t}\right)}{2}\right] + \mathrm{j}\tanh\left[\frac{L\left(c_{2i,t}\right)}{2}\right]\right\}. \tag{9}$$

The suboptimal simplified log-likelihood ratios are

$$\hat{\Lambda}(c_{i,t}) = \ln \frac{\sum\limits_{x=f_a(c_a);c_{ai}=1} \exp\left\{-\frac{\left|\tilde{y}_{a,t}^{(k)} - x\right|^2}{\left(\mathbf{K}^{(k)}\right)_{aa}}\right\}}{\sum\limits_{x=f_a(c_a);c_{ai}=1} \exp\left\{-\frac{\left|\tilde{y}_{a,t}^{(k)} - x\right|^2}{\left(\mathbf{K}^{(k)}\right)_{aa}}\right\}}, \tag{10}$$

where $a = 1 + \left\lfloor \frac{i-1}{M} \right\rfloor$ and matrix $\mathbf{K}^{(k)}$ can be approximated as

$$\mathbf{K}^{(k)} \cong \tilde{\mathbf{K}}^{(k)} = \frac{1}{N}\tilde{\mathbf{Y}}^{(k)}\tilde{\mathbf{Y}}^{(k)H} - E_S\mathbf{I}_{N_t}\,. \tag{11}$$

3. Asymmetric Turbo Codes for Antenna Diversity Systems

An asymmetric turbo code is composed of two recursive convolutional codes with different generator polynomials (Takeshita *et al.*, 1999). In order to improve the BER, the cited authors have supposed that one of the component codes has a non-primitive feedback polynomial (“weak”), and the second code, a primitive feedback polynomial (“strong”). The weak component code leads to the improvement of the BER at low SNR values, while the strong component code, at high SNR values, is responsible for creating a larger minimum distance of the asymmetric turbo code. In this study different combinations of primitive and non-primitive polynomials are used. The primitive polynomial leads to a maximum cycle length in the states diagram.

We denote with $C_T[c_1, c_2]$ the turbo codes which have parallel concatenated two systematic recursive convolutional codes (SRCC), c_1 and c_2. The first trellis is terminated and second one is not. The convolutional codes will be denoted by (FF_{oct}, FB_{oct}), where FF_{oct} represents the feed forward encoding polynomial and FB_{oct} is the feedback encoding polynomial. We have studied both cases, when the feedback polynomial is primitive and non-primitive, respectivety.

For a QPSK modulation, the simulations are performed for a turbo code having the global coding rate of 1/2, with puncturing, over a block Rayleigh fading channel. We have used the genie stopper criterion for stopping the iteration, meaning that the iterations in turbo decoding are stopped when the decoded bit frame is identical to the information bit frame originally coded.

From former simulations it has been shown that the feedback polynomial must be primitive, in order that the effective free distance (the minimum distance obtained for the input sequence of weight 2) to be high. This applies to AWGN channel.

In Table 1 the component convolutional codes are presented for the turbo codes used in the scheme in Fig. 1. *Px* denotes the primitive generator polynomial of the *x*-state component code and *NPx* denotes the non-primitive generator polynomial of the *x* state component code.

Table 1

Asymmetric Turbo Code	Short notation
C_T[(5,7) , (15,13)]	*P*4-*P*8
C_T[(15,13) , (5, 7)]	*P*8-*P*4
C_T[(5,7) , (15,17)]	*P*4-*NP*8
C_T[(7,5) , (15,13)]	*NP*4-*P*8
C_T[(7,5) , (15,17)]	*NP*4-*NP*8

4. Simulation Results

The simulations were performed considering the same pattern as Biglieri *et al*. (2005), using QPSK modulation (M = 2). A Ω'-QPP interleaver of length 2,080 is used in the turbo code of rate 1/2. The free term for the Ω'-QPP interleaver is found by maximizing the corner merit because only the first trellis is terminated. The simulations were also performed for a max-log-APP turbo decoding algorithm in a system using a spatial interference canceling interface (iterative MMSE receiver).

Different forward and feedback generator polynomials of memory 2 and 3 are used, and also a random interleaver between the turbo encoder and the serial to parallel converter. A spectral efficiency of $\eta = 16$ bits/sec/Hz is obtained, because 16 transmission and 16 reception antennas are considered. The space time codeword is a matrix with 130 columns. The turbo decoder performs 10 iterations with a genie stopper type stop criterion as was mentioned in Section **3**. To cancel spatial interferences, a number of k = 0, k = 1 or k = 4 iterations were used.

After the analysis of the influence of the extrinsic information scaling coefficient, denoted with s (Trifina *et al*., 2011), that ranges from 0.6 to 1 with the step 0.05, we considered in this paper, for k = 0 , s = 0.9, for k = 1, s = 0.8 and for k = 4 , s = 0.75.

Simulation results showing FER and BER curves *versus* signal-to-noise ratio per bit (E_b/N_0) for k = 0 are given in Fig. 3. Fig. 4 presents simulations results for k = 1, and Fig. 5, for k = 4.

From these simulations it can be seen that the codes with primitive feedback polynomials, *P*4-*P*8 and *P*8-*P*4, with similar performances, lead to better FER and BER results, compared to those with a non-primitive feedback polynomial. In FER performance order, the best performances are obtained by the codes with primitive feedback polynomials, *P*4-*P*8, *P*8-*P*4, *P*4-*NP*8 and

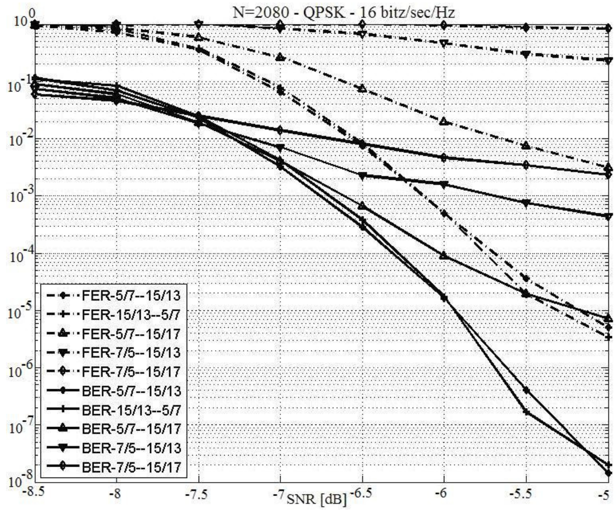

Fig. 3 – Performances for $k = 0$.

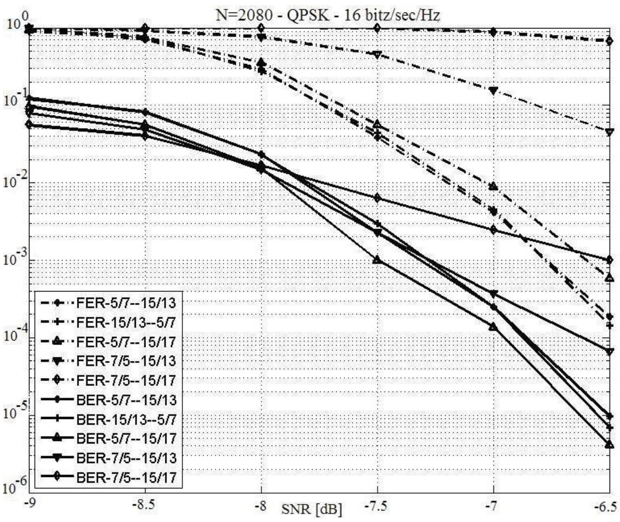

Fig. 4 – Performances for $k = 1$.

*NP*4-*P*8, and the *NP*4-*NP*8 combination leading to considerably lower performances. Also, we note the influence of the primitive feedback polynomial code with memory 2 on the performances. In BER, the best performances are also given by the first three codes, and the last two ones have lower performances.

As k increases, the FER and BER performances are better (Biglieri *et al.*, 2005), but for k = 4 and k = 1, the double iterations introduce relatively more errors compared to k = 0. As an observation, for k = 0, from particular simulations it can be seen an improvement in FER and BER performances when F is increased, because if the noise affects a block, all the contained bits are

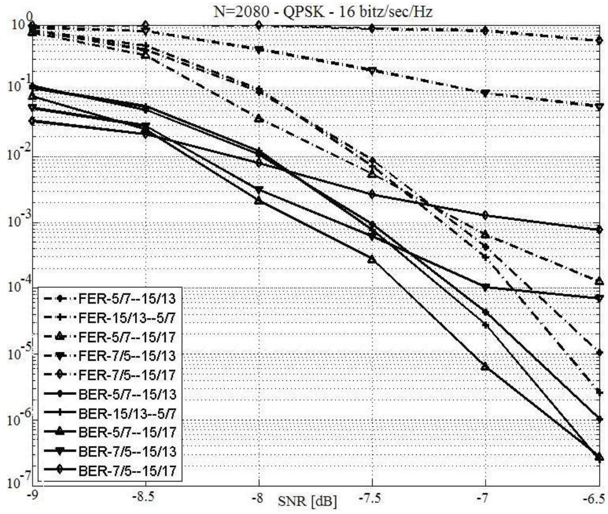

Fig. 5 – Performances for k = 4.

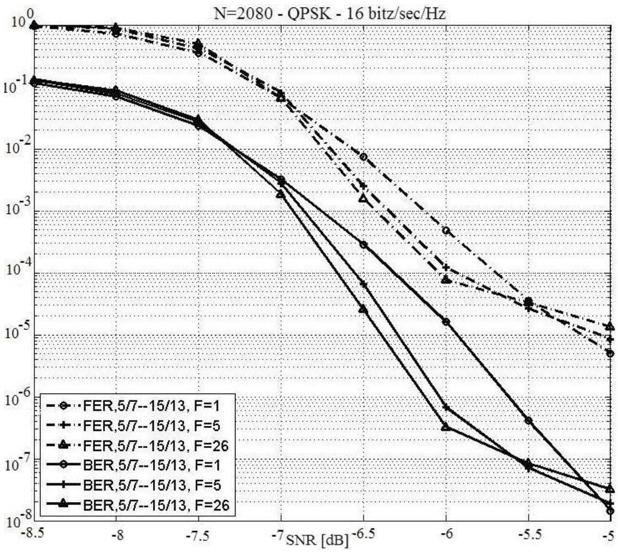

Fig. 6 – Performances for k = 0, different F.

affected and if a block is correctly demodulated, none of the contained bits is affected. For k = 0, and for both component convolutional codes with primitive feedback polynomials, $P4$-$P8$, $P8$-$P4$, the simulations were made considering F = 1, F = 5 and F = 26.

With a $P4$-$P8$ and a $P8$-$P4$ turbo code, for F = 26, at FER = 10^{-4} dB, was obtained a coding gain of 0.4 dB, respectively 0.45 dB, compared to F = 1, and a 0.1 dB coding gain compared to F = 5. In BER performances, if F = 26, at BER = 10^{-6} dB, a 0.5 dB, respectively 0.6 dB coding gain is obtained compared to F = 1, and 0.15 dB, respectively 0.2 dB coding gain, compared to F = 5.

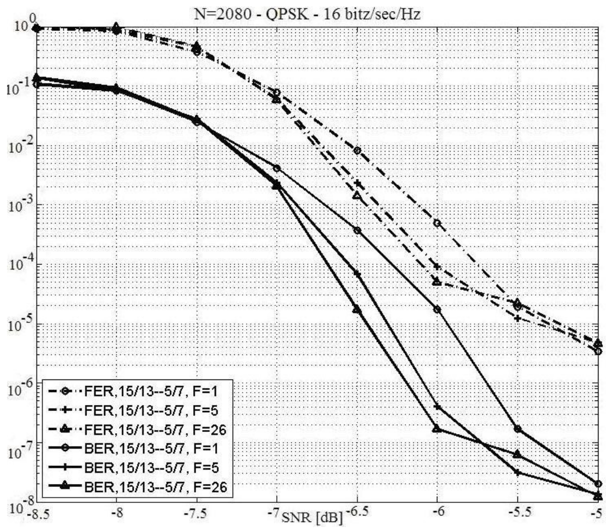

Fig. 7 – Performances for k = 0, different F.

5. Conclusions

The performances of asymmetric turbo codes on a quasi-static fading channel in a limited-complexity double iterative decoder are analysed. Using a Ω'-QPP interleaver, the simulations were performed for a max-log-APP turbo decoding algorithm in a system using a spatial interference canceling interface (iterative MMSE receiver). In the doubly-iterative decoding process, scaling both the extrinsic information of the turbo decoder and the information at the input of the interference canceling block is used. k = 0, k = 1 or k = 4 iterations were used to cancel spatial interferences. The simulations were performed for a scaling coefficient of s = 0.9 for k = 0, s = 0.8 for k = 1, and s = 0.75 for k = 4, as is shown by Trifina *et al.*, (2011).

The turbo code component codes are not identical. The memory of the component encoders is 2 and 3 for primitive and non-primitive feedback polynomials. The codes with primitive feedback polynomials lead to better results and those with memory 2 have a higher influence on the system performance when they are upper codes in turbo codes. That is because only the first trellis in the turbo code is terminated and second one is not.

From particular simulations it can be seen that when F is increased at the values F = 5 and F = 26, the FER and BER performances are improved, compared to the case when F = 1.

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MODULAȚIE CODATĂ TURBO ASIMETRICĂ PENTRU SISTEME MIMO FOLOSIND ALGORITM DE DECODARE TURBO DUBLU ITERATIV

(Rezumat)

Se analizează performanțele modulației codate turbo utilizând coduri turbo asimetrice cu diversitate de antene la emisie şi recepție. S-au considerat coduri convoluționale componente asimetrice de memorie 2 şi 3 (adică cu 4 şi 8 stări), iar polinoamele lor generatoare de reacție sunt atât primitive cât şi neprimitive. Pentru a studia ambele cazuri, au fost realizate simulări pentru a obține rata erorii de bit (BER) şi rata erorii de cadru (FER). Din rezultatele simulărilor s-a observat că acele coduri care sunt construite doar după polinoame de reacție primitive conduc la performanțe FER mai bune, pe când cele care sunt construite după polinoame de reacție neprimitive conduc la mici îmbunătățiri ale BER în domeniul SNR redus.